Iterated transforms of the form $F_1(F_2(\ldots))$ where $F_i(z) = \int_0^\infty \frac{d\xi_i(\beta)}{1+z\beta}$

$$f(z)=\int_0^\infty \frac{d\xi(s)}{1+zs} \qquad z^{-1}f(z^{-1})\ldots$$

$$f(g(z))=\int_0^\infty \frac{d\xi(s)}{1+s\int_0^\infty \frac{d\Delta(s')}{1+zs'}} \qquad 1+z_0\alpha>\lambda_0\beta'$$

$S(-H(z))$ analytic for $|z|<\frac{\lambda_0\beta'-1}{\alpha}$

$$z^{-1}f(g(z^{-1}))=\int_0^\infty \frac{d\xi(s)}{z+s\int_0^\infty \frac{d\Delta(s')}{z+s'}}$$

Λ

$$\mathrm{Im}(z)>0 \quad \mathrm{Im}(z^{-1}f(z^{-1}))<0$$

$$f(-g(z))=\int_0^\infty \frac{d\xi(s)}{1-s\int_0^\infty \frac{d\Delta(s')}{1+zs'}} = F(z)$$

$$z^{-1}F(z)=\int_0^\infty \frac{d\xi(s)}{z-s\int_0^\infty \frac{d\Delta(s')}{z+s'}}$$

$z_0>-\beta^{-1}$

$\frac{\lambda_0}{1+z_0\alpha}$

$\int_\alpha^{\beta'}$

$$\mathrm{Im}(z)>0 \quad \mathrm{Im}\int_0^\infty \frac{d\Delta(s')}{z+s'}<0 \qquad \frac{\lambda_0}{1+z_0\alpha}\beta'<1$$

$S(z)$ of Stieltjes type $H(z)$ of Hamburger type

$S(-H(z))$ of Hamburger Nevanlinna type.

$H(z)$ of Stieltjes type $\Lambda(\beta)-\Lambda(\alpha)=\lambda_0$ $\int_\alpha^\beta$ α^{-1} $-\beta^{-1}$ z_0

must look for max ve $\frac{\lambda_0}{z_0+\beta^{-1}}$

$F \in S$: $F(\lambda) = \int_0^\infty \frac{d\xi(s)}{\lambda + s}$ ξ bdd nondecr in $0 \le s \le \infty$

$F \in H$: .. $\int_{-\infty}^{\infty}$ $-\infty \le s \le \infty$

$$\cancel{F(\lambda)}\ F_1(F_2(\lambda)^{-1}) = \int_0^\infty \frac{d\xi_1(s)}{\dfrac{1}{\int_0^\infty \frac{d\xi_2(s')}{\lambda + s'}} + s}$$

$Im(\lambda) > 0$ $Im \int_0^\infty \frac{d\xi_2(s')}{\lambda + s'} \le 0$, $Im \frac{1}{\int ..} \ge 0$ Im

$$\frac{\int_0^\infty d\xi_1(s_1)}{s_1 +} \quad \frac{\int_0^\infty d\xi_2(s_2)}{s_2 +} \cdots$$

$0 < \lambda_0 \lneq \lambda_1 \lneq \lambda_2 \to \infty$ $F(\lambda_0) > F(\lambda_1) > F(\lambda_2)$

$$C_n = \frac{\int_{\alpha_1}^{\beta_1} d\xi_1(s)}{s_1 +} \quad \frac{\int_{\alpha_2}^{\beta_2} d\xi_2(s_2)}{s_2 +} \cdots \frac{\int_{\alpha_n}^{\beta_n} \xi_n(s_n)}{s_n + y}$$

$$= C_n[\Xi; A, B; \lambda]$$

$C_{2n+2} > C_{2n}$ $\quad C_{2n+1} \lneq C_{2n-1}$

$C_{2n} < C_{2n+2} \le \; < C_{2n+1} < C_{2n-1}$ $\quad C_{2n}$

$$C_{2n+1} = \frac{\int_{\alpha_1}^{\beta_1} d\xi_1(s)}{s_1 + \dfrac{1}{C_{2n}[\Xi^{(1)}; A^{(1)}, B^{(1)}; \lambda]}}$$

$$C_n^{(m)} = \frac{\int_{\alpha_m}^{\beta_m} d\xi_m}{s_m + \dfrac{1}{\dfrac{\int_{\alpha_{m+1}}^{\beta_{m+1}} d\xi_{m+1}}{s_{m+1} + \dfrac{1}{\quad}}}}$$

$$C_n^{(m)} = \frac{\int_{\alpha_m}^{\beta_m} d\xi_m(s_m)}{s_m +} \; 1 \Big/ \frac{\int_{\alpha_{m+1}}^{\beta_{m+1}} d\xi_{m+1}(s_{m+1})}{s_{m+1} +} \cdots 1 \Big/ \frac{\int_{\alpha_{m+n-1}}^{\beta_{m+n-1}} d\xi_{m+n-1}(s_{m+n-1})}{s_{m+n-1} + \lambda}$$

$$C_n^{(m)} = F_m\left(1/C_{n-1}^{(m+1)}\right)$$

$$C_1^{(m)} = \frac{\int_{\alpha_m}^{\beta_m} d\xi_m(s_m)}{s_m + \lambda} \qquad C_2 = \frac{\int_{\alpha_m}^{\beta_m} d\xi_m(s_m)}{s_m + \dfrac{1}{\int_{\alpha_{m+1}}^{\beta_{m+1}} \dfrac{d\xi_{m+1}(s_{m+1})}{s_{m+1} + \lambda}}}$$

$$\int_{\alpha_m}^{\beta_m} d\xi_m(s_m) = t^{(m)}$$

$$\int_{\alpha_{m+1}}^{\beta_{m+1}} \frac{d\xi_{m+1}(s_{m+1})}{s_{m+1} + \lambda} < \frac{t^{(m+1)}}{\lambda + \alpha_{m+1}} \qquad \frac{1}{\int_{\alpha_{m+1}}^{\beta_{m+1}} \dfrac{d\xi_{m+1}(s_{m+1})}{s_{m+1} + \lambda}} > \frac{\lambda + \alpha_{m+1}}{t^{(m+1)}}$$

$$C_2^{(m)} < \int_{\alpha_m}^{\beta_m} \frac{d\xi_m(s_m)}{s_m + \cancel{\dfrac{t^{(m+1)}}{\lambda + \alpha_{m+1}}} \dfrac{\lambda + \alpha_{m+1}}{t^{(m+1)}}}$$

$$< \frac{t_m}{\cancel{\alpha_m + \dfrac{\lambda + \alpha_{m+1}}{t^{(m+1)}}}} \qquad C_2^{(m)} < F_m\left(\frac{\lambda + \alpha_{m+1}}{t^{(m+1)}}\right)$$

$$< \frac{t_m}{\alpha_m + \dfrac{\lambda + \alpha_{m+1}}{t_{m+1}}}$$

$$C_3^{(m)} = \frac{\int_{\alpha_m}^{\beta_m} d\xi_m(\lambda)}{s_m + \dfrac{1}{C_2^{(m+1)}}}$$

$$\cancel{\frac{1}{C_2^{(m+1)}} > \frac{\alpha_m + \dfrac{\lambda + a_{m+2}}{t^{m+2}}}{t_m}}$$

$$\cancel{\frac{1}{C_2^{(m+1)}} > F_m\left(\frac{\lambda + \alpha_{m+2}}{t^{(m+2)}}\right)}$$

$d\xi_m = a_n$ at $s_m = 1$

$$\frac{1}{C_2^{(m+1)}} > \frac{1}{F_{m+1}\left(\dfrac{\lambda + \alpha_{m+2}}{t^{(m+2)}}\right)}$$

$$\frac{a_1}{1 + \dfrac{1}{\dfrac{a_{m+1}}{1 + \dfrac{1}{\dfrac{a_{m+2}}{1 + \dfrac{1}{}}}}}}$$

$$F_{m+1}(\lambda') < \frac{t_{m+1}}{\lambda' + \alpha_{m+1}}$$

$$\frac{1}{F_{m+1}(\lambda')} > \frac{\lambda' + \alpha_{m+1}}{t_{m+1}}$$

$$\frac{1}{F_{m+1}\left(\dfrac{\lambda + \alpha_{m+2}}{t_{m+2}}\right)} > \frac{\dfrac{\lambda + \alpha_{m+2}}{t_{m+2}} + \alpha_{m+1}}{t_{m+1}}$$

$$\frac{1}{C_2^{(m+1)}} > \frac{\lambda + \alpha_{m+2} + \alpha_{m+1} t_{m+2}}{t_{m+1} t_{m+2}}$$

$$C_3^{(m)} < \frac{t_m}{\alpha_m + \dfrac{\lambda + \alpha_{m+2} + \alpha_{m+1} t_{m+2}}{t_{m+1} t_{m+2}}}$$

a_1 ; $\dfrac{a_1}{1+\dfrac{1}{a_2}}$ $\dfrac{a_1}{b_1+\dfrac{a_2}{b_2}}$ Im saltim am at $g_n = 1$

$$\cfrac{a_1}{1+\cfrac{1}{a_2+\cfrac{1}{1+\cfrac{1}{a_3}}}}$$

$$\cfrac{\frac{1}{b_1}}{\frac{\cdot}{a_1}} + \cfrac{a_2/b_1}{b_2}$$

$$\cfrac{a_0 b_1}{b_0 b_1 + a_1 - \cfrac{a_1 a_2 b_3}{(b_1 b_3 + a_2) b_3 + b_1 a_3 -}}$$

$$\cfrac{a_n}{1+\cfrac{1}{0+\cfrac{a_{n+2n}}{1+\cfrac{1}{0+\cfrac{a_{n+2}}{1+\cdots}}}}}$$

$$b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2+\cfrac{a_3}{b_3}}} \sim b_0 + \frac{a_1}{b_1} - \frac{a_1 a_2 b_3 b_1^{-1}}{(b_1 b_3 + a_2 b_3) + b_1 a_3}$$

$$\cfrac{a_1}{b_1 + \cfrac{a_2 b_3}{b_2 b_3 + a_3}} = \frac{a_1(b_2 b_3 + a_3)}{b_1(b_2 b_3 + a_3) + a_2 b_3}$$

$$\frac{a_1 (b_1 b_3 + \cancel{a_2 b_3}) + a_1 b_1 a_3 - \cancel{a_1 a_2 b_3}}{b_1\{(b_1 b_3 + a_2 b_3) + b_1 a_3\}}$$

$$\frac{b_0 b_1 + a_1}{b_1} - \frac{a_1 a_2 \frac{b_3}{b_1}}{(b_1 b_2 + a_2) b_3 + b_1 a_3 -} \cdots \frac{a_{2\nu-1} a_{2\nu} b_{2\nu-3} b_{2\nu+1}}{(b_{2\nu-1} b_{2\nu} + a_{2\nu}) b_{2\nu+1} + b_{2\nu-1} a_{2\nu+1}}$$

$b_0' = 0 \quad b_1' = 1 \quad a_1' = a_1 \quad a_{2\nu}' = 1 \quad a_{2\nu-1}' = a_\nu$

$b_{2\nu}' = 0 \quad b_{2\nu-1}' = 1$

$$a_1 - \frac{a_1}{1 + a_2 -} \; \frac{a_2 . 1 . 1}{1 + 1 . a_3 -} \cdots \frac{a_\nu}{1 + a_{\nu+1} -}$$

$$a_1 - \frac{a_1}{1 + a_2 -} \; \frac{a_2}{1 + a_3 -} \cdots \frac{a_\nu}{1 + a_{\nu+1} -} \cdots$$

$$\frac{1}{0 +} \; \frac{a_1}{1 +} \; \frac{1}{0 +} \; \frac{a_2}{1 +} \; \frac{1}{0 +} \cdots$$

$a_{2\nu-1}' = 1 \quad a_{2\nu}' = a_\nu \quad b_{2\nu-1}' = 0 \quad b_{2\nu}' = 1$

$$\frac{1}{a_1 -} \qquad \frac{a_\nu 1 .}{1 . 1 + a_{\nu+1}}$$

$A_{-1} = 1 \quad B_{-1} = 0 \quad A_0 = a_1 \quad B_0 = 1 \quad A_1 = a_1 a_2 \quad B_1 = 1 + a_2$

$A_2 = (1 + a_3) a_1 a_2 - a_2 a_1 = a_1 a_2 a_3$

$B_2 = (1 + a_3)(1 + a_2) - a_2 = 1 + a_3 + a_2 a_3$

$A_3 = (1 + a_4) a_1 a_2 a_3 - a_3 a_1 a_2 = a_1 a_2 a_3 a_4$

$B_3 = (1 + a_4)\{1 + a_3 + a_2 a_3\} - a_3(1 + a_2)$

$1 + \cancel{a_3} + \cancel{a_2 a_3} + a_4 + a_3 a_4 + a_2 a_3 a_4 - \cancel{a_3} - \cancel{a_2 a_3}$

$$A_n = \prod_{\tau=1}^{n+1} a_\tau \quad B_n = \sum_{\nu=0}^{n} \prod_{\tau=\nu+1}^{n} a_{\tau+1}$$

$\tau+1=\tau'$
$\tau=n \quad \tau'=n+1$
$\tau=\nu+1 \quad \tau'=\nu+2$

$$= \sum_{\nu=0}^{n} \prod_{\tau=\nu+2}^{n+1} a_\tau$$

$$A_{n+1} = (1+a_{n+2}) \prod_{\tau=1}^{n+1} a_\tau - a_{n+1} \prod_{\tau=1}^{n} a_\tau = a_{n+2} \prod_{\tau=1}^{n+1} a_\tau$$

$$B_{n+1} = (1+a_{n+2}) \sum_{\nu=0}^{n} \prod_{\tau=\nu+2}^{n+1} a_\tau - a_{n+1} \sum_{\nu=0}^{n-1} \prod_{\tau=\nu+2}^{n} a_\tau$$

$$= \sum_{\nu=0}^{n} \prod_{\tau=\nu+2}^{n+1} a_\tau + \sum_{\nu=0}^{n} \prod_{\tau=\nu+2}^{n+2} a_\tau - \sum_{\nu=0}^{n-1} \prod_{\tau=\nu+2}^{n+1} a_\tau$$

$$= 1 + \sum_{\nu=0}^{n} \prod_{\tau=\nu+2}^{n+2} a_\tau = \sum_{\nu=0}^{n+1} \prod_{\tau=\nu+2}^{n+2} a_\tau$$

$$\frac{A_n}{B_n} = \frac{\prod_{\tau=1}^{n+1} a_\tau}{\sum_{\nu=0}^{n} \prod_{\tau=\nu+2}^{n+1} a_\tau} = \left\{ \sum_{\nu=0}^{n} \prod_{\tau=1}^{\nu+1} a_\tau^{-1} \right\}^{-1}$$

$$= \left\{ \frac{1}{a_1} + \frac{1}{a_1 a_2} + \frac{1}{a_1 a_2 a_3} + \cdots \frac{1}{a_1 a_2 a_3 \ldots a_{n+1}} \right\}^{-1}$$

convergence if for some ν' $\frac{1}{a_\nu} < \eta < 1$

i.e. $a_\nu > \eta' > 1$. converge to finite non real number

if $a_\nu < \eta < 1$ " " zero

$$\cfrac{a_m}{\beta s_m + \cfrac{1}{\cfrac{a_2}{s_2+}}} \quad \cdots \quad \frac{a_1}{s_1+}\,\frac{1}{0+}\,\frac{a_2}{s_2+}\,\frac{1}{0+}\cdots$$

$b'_0 = 0 \quad b'_1 = s_1 \quad a'_{2\nu} = 1 \quad a'_{2\nu-1} = a_\nu$

$b'_{2\nu} = 0 \quad b'_{2\nu-1} = s_\nu$

$$\frac{a_1}{s_1} - \frac{a_1\cdot \frac{s_2}{s_1}}{s_2 + s_1 a_2 -} \cdots \frac{a_\nu\cdot 1\cdot s_{\nu-1}s_{\nu+1}}{s_{\nu+1} + s_\nu a_{\nu+1} -}$$

$$\frac{a_1}{s_1}\left\{1 - \frac{s_2}{s_2 + s_1 a_2 -}\ \frac{a_2 s_1 s_3}{s_3 + s_2 a_3 -}\cdots\right.$$

$A_0 = 1 \quad B_0 = 1 \quad A_1 = s_1 a_2 \quad B_2 = s_2 + s_1 a_2$

$A_2 = (s_3 + s_2 a_3) s_1 a_2 - a_2 s_1 s_3 \cdot 1 = s_1 s_2 a_2 a_3$

$B_2 = (s_3 + s_2 a_3)(s_2 + s_1 a_2) - a_2 s_1 s_3 \cancel{(s_2 + s_1 a_2)}\, 1$

$= s_2 s_3 + s_2^2 a_3 + s_1 s_2 a_2 a_3$

$B_3 = (s_4 + s_3 a_4)\{s_2 s_3 + s_2^2 a_3 + s_1 s_2 a_2 a_3\}$

$- a_3 s_2 s_4 (s_2 + s_1 a_2)$

$s_2 s_3 s_4 + \cancel{s_2^2 s_4 a_3} + \cancel{s_1 s_2 s_4 a_2 a_3}$

$+ s_2 s_3^2 a_4 + s_2^2 s_3 a_3 a_4 + s_1 s_2 s_3 a_2 a_3 a_4 - \cancel{s_2^2 s_4 a_3} - \cancel{s_1 s_2 s_4 a_2 a_3}$

$s_2 s_3 s_4 + s_2 s_3^2 a_4 + s_2^2 s_3 a_3 a_4 + s_1 s_2 s_3 a_2 a_3 a_4$

$$B_4 = (s_5 + s_4 a_5)\{s_2 s_3 s_4 + s_2 s_3^2 a_4 + s_2^2 s_3 a_3 a_4 + s_1 s_2 s_3 a_2 a_3 a_4\}$$

$$- a_4 s_3 s_5 \{s_2 s_3 + s_2^2 a_3 + s_1 s_2 a_2 a_3\}$$

$$= s_2 s_3 s_4 s_5 + s_2 s_3 s_4^2 a_5 + s_2 s_3^2 s_4 a_4 a_5$$

$$+ s_2^2 s_3 s_4 a_3 a_4 a_5 +$$

$$\cancel{B_n = \sum_{\nu=0}^{n}\Big\{\prod_{\tau=n-\nu}}$$

$$B_n = \Big\{\prod_{\tau=2}^{n} s_\tau\Big\}\sum_{\nu=0}^{n}\Big\{s_{\nu+1}\prod_{\tau=\nu+2}^{n+1} a_\tau\Big\}$$

$$A_n = \Big\{\prod_{\tau=2}^{n} s_\tau\Big\}\prod_{\tau=1}^{n+1} a_\tau$$

$$B_{n+1} = (s_{n+2} + s_{n+1}a_{n+2})\Big\{\prod_{\tau=2}^{n} s_\tau\Big\}\sum_{\nu=0}^{n}\Big\{s_{\nu+1}\prod_{\tau=\nu+2}^{n+1} a_\tau\Big\}$$

$$- a_{n+1} s_n s_{n+2}\Big\{\prod_{\tau=2}^{n-1} s_2\Big\}\sum_{\nu=0}^{n-1}\Big\{s_{\nu+1}\prod_{\tau=\nu+2}^{n} a_\tau\Big\}$$

$$= s_{n+1}a_{n+2}\Big\{\prod_{\tau=2}^{n} s_\tau\Big\}\sum_{\nu=0}^{n}\Big\{s_{\nu+1}\prod_{\tau=\nu+2}^{n+1} a_\tau\Big\}$$

$$+ s_{n+2}\prod_{\tau=2}^{n} s_\tau\{s_{n+1}\} = B_{n+1} \text{ from above}$$

$$A_{n+1} = (g_{n+2}+g_{n+1}a_{n+2})\left\{\prod_{\tau=2}^{n} g_\tau\right\}\prod_{\tau=1}^{n+1} a_\tau$$

$$-a_{n+1}g_n g_{n+2}\left\{\prod_{\tau=2}^{n-1} g_\tau\right\}\prod_{\tau=1}^{n} a_\tau = A_{n+1} \text{ from above}$$

$$\frac{A_n}{B_n} = \left\{\sum_{\nu=0}^{n} g_{\nu+1}\prod_{\tau=1}^{\nu+1} a_\tau^{-1}\right\}^{-1}$$

$$= \sum_{\nu=1}^{n+1}\left\{g_\nu\prod_{\tau=1}^{\nu+1} a_\tau^{-1}\right\}^{-1}$$

$$\frac{g_{\nu+1}}{g_\nu}\frac{1}{a_{\nu+1}} < \eta < 1 \quad \text{i.e } g_\nu a_{\nu+1}\eta > g_{\nu+1} \quad \nu \geq \mathcal{J}_{\nu'}$$

for some $\eta < 1$

then convergence to finite +ve real number

if $g_\nu a_{\nu+1}\eta < g_{\nu+1}$ for some $\eta > 1$ & $\nu \geq \mathcal{J}_{\nu'}$

then converge to zero.