LEARNING OBJECTIVES

In this section, you will:

- Identify power functions.
- Identify polynomial functions.
- Identify the degree and leading coefficient of a polynomial function

MODULE 2.1 - INTRODUCTION TO POWER FUNCTIONS AND POLYNOMIAL FUNCTIONS

Figure 1 (credit: Jason Bay, Flickr)

Suppose a certain species of bird thrives on a small island. Its population over the last few years is shown in **Table 1**.

Year	2009	2010	2011	2012	2013
Bird Population	800	897	992	1,083	1,169

Table 1

The population can be estimated using the function $P(t) = -0.3t^3 + 97t + 800$, where $P(t)$ represents the bird population on the island t years after 2009. We can use this model to estimate the maximum bird population and when it will occur. We can also use this model to predict when the bird population will disappear from the island. In this section, we will examine functions that we can use to estimate and predict these types of changes.

Identifying Power Functions

Before we can understand the bird problem, it will be helpful to understand a different type of function. A **power function** is a function with a single term that is the product of a real number, a **coefficient**, and a variable raised to a fixed real number. (A number that multiplies a variable raised to an exponent is known as a coefficient.)

As an example, consider functions for area or volume. The function for the area of a circle with radius r is

$$A(r) = \pi r^2$$

and the function for the volume of a sphere with radius r is

$$V(r) = \frac{4}{3}\pi r^3$$

Both of these are examples of power functions because they consist of a coefficient, π or $\frac{4}{3}\pi$, multiplied by a variable r raised to a power.

> ***power function***
> A **power function** is a function that can be represented in the form
> $$f(x) = kx^p$$
> where k and p are real numbers, and k is known as the **coefficient**.

Q & A…

Is $f(x) = 2^x$ a power function?

No. A power function contains a variable base raised to a fixed power. This function has a constant base raised to a variable power. This is called an exponential function, not a power function.

Example 1 Identifying Power Functions

Which of the following functions are power functions?

$f(x) = 1$	Constant function
$f(x) = x$	Identify function
$f(x) = x^2$	Quadratic function
$f(x) = x^3$	Cubic function
$f(x) = \dfrac{1}{x}$	Reciprocal function
$f(x) = \dfrac{1}{x^2}$	Reciprocal squared function
$f(x) = \sqrt{x}$	Square root function
$f(x) = \sqrt[3]{x}$	Cube root function

Solution All of the listed functions are power functions.

The constant and identity functions are power functions because they can be written as $f(x) = x^0$ and $f(x) = x^1$ respectively.

The quadratic and cubic functions are power functions with whole number powers $f(x) = x^2$ and $f(x) = x^3$.

The reciprocal and reciprocal squared functions are power functions with negative whole number powers because they can be written as $f(x) = x^{-1}$ and $f(x) = x^{-2}$.

The square and cube root functions are power functions with fractional powers because they can be written as $f(x) = x^{1/2}$ or $f(x) = x^{1/3}$.

Try It #1

Which functions are power functions?

$$f(x) = 2x^2 \cdot 4x^3 \qquad g(x) = -x^5 + 5x^3 \qquad h(x) = \frac{2x^5 - 1}{3x^2 + 4}$$

Identifying Polynomial Functions

An oil pipeline bursts in the Gulf of Mexico, causing an oil slick in a roughly circular shape. The slick is currently 24 miles in radius, but that radius is increasing by 8 miles each week. We want to write a formula for the area covered by the oil slick by combining two functions. The radius r of the spill depends on the number of weeks w that have passed. This relationship is linear.

$$r(w) = 24 + 8w$$

We can combine this with the formula for the area A of a circle.

$$A(r) = \pi r^2$$

Composing these functions gives a formula for the area in terms of weeks.

$$\begin{aligned} A(w) &= A(r(w)) \\ &= A(24 + 8w) \\ &= \pi(24 + 8w)^2 \end{aligned}$$

Multiplying gives the formula.

$$A(w) = 576\pi + 384\pi w + 64\pi w^2$$

This formula is an example of a **polynomial function**. A polynomial function consists of either zero or the sum of a finite number of non-zero terms, each of which is a product of a number, called the coefficient of the term, and a variable raised to a non-negative integer power.

> ***polynomial functions***
> Let n be a non-negative integer. A **polynomial function** is a function that can be written in the form
> $$f(x) = a_n x^n + \ldots + a_2 x^2 + a_1 x + a_0$$
> This is called the general form of a polynomial function. Each a_i is a coefficient and can be any real number, but a_n cannot $= 0$. Each expression $a_i x^i$ is a **term of a polynomial function**.

Example 2 **Identifying Polynomial Functions**

Which of the following are polynomial functions?

$$f(x) = 2x^3 \cdot 3x + 4 \qquad g(x) = -x(x^2 - 4) \qquad h(x) = 5\sqrt{x + 2}$$

Solution The first two functions are examples of polynomial functions because they can be written in the form $f(x) = a_n x^n + \ldots + a_2 x^2 + a_1 x + a_0$, where the powers are non-negative integers and the coefficients are real numbers.

- $f(x)$ can be written as $f(x) = 6x^4 + 4$.
- $g(x)$ can be written as $g(x) = -x^3 + 4x$.
- $h(x)$ cannot be written in this form and is therefore not a polynomial function.

Identifying the Degree and Leading Coefficient of a Polynomial Function

Because of the form of a polynomial function, we can see an infinite variety in the number of terms and the power of the variable. Although the order of the terms in the polynomial function is not important for performing operations, we typically arrange the terms in descending order of power, or in general form. The **degree** of the polynomial is the highest power of the variable that occurs in the polynomial; it is the power of the first variable if the function is in general form. The **leading term** is the term containing the highest power of the variable, or the term with the highest degree. The **leading coefficient** is the coefficient of the leading term.

terminology of polynomial functions

We often rearrange polynomials so that the powers are descending.

$$f(x) = \underbrace{\overset{\text{Leading coefficient}}{a_n} x^{\overset{\text{Degree}}{n}}}_{\text{Leading term}} + \dots + a_2x^2 + a_1x + a_0$$

When a polynomial is written in this way, we say that it is in general form.

How To...

Given a polynomial function, identify the degree and leading coefficient.

1. Find the highest power of x to determine the degree function.
2. Identify the term containing the highest power of x to find the leading term.
3. Identify the coefficient of the leading term.

Example 5 Identifying the Degree and Leading Coefficient of a Polynomial Function

Identify the degree, leading term, and leading coefficient of the following polynomial functions.

$$f(x) = 3 + 2x^2 - 4x^3$$
$$g(t) = 5t^5 - 2t^3 + 7t$$
$$h(p) = 6p - p^3 - 2$$

Solution For the function $f(x)$, the highest power of x is 3, so the degree is 3. The leading term is the term containing that degree, $-4x^3$. The leading coefficient is the coefficient of that term, -4.

For the function $g(t)$, the highest power of t is 5, so the degree is 5. The leading term is the term containing that degree, $5t^5$. The leading coefficient is the coefficient of that term, 5.

For the function $h(p)$, the highest power of p is 3, so the degree is 3. The leading term is the term containing that degree, $-p^3$; the leading coefficient is the coefficient of that term, -1.

Try It #3

Identify the degree, leading term, and leading coefficient of the polynomial $f(x) = 4x^2 - x^6 + 2x - 6$.

INTRODUCTION TO POWER FUNCTIONS AND POLYNOMIAL FUNCTIONS EXERCISES

VERBAL

1. Explain the difference between the coefficient of a power function and its degree.

ALGEBRAIC

For the following exercises, identify the function as a power function, a polynomial function, or neither.

2. $f(x) = x^5$

3. $f(x) = (x^2)^3$

4. $f(x) = x - x^4$

5. $f(x) = \dfrac{x^2}{x^2 - 1}$

6. $f(x) = 2x(x + 2)(x - 1)^2$

7. $f(x) = 3^{x+1}$

For the following exercises, find the degree and leading coefficient for the given polynomial.

8. $-3x$

9. $7 - 2x^2$

10. $2x^2 - 3x^5 + x - 6$

11. $x(4 - x^2)(2x + 1)$

12. $x^2(2x - 3)^2$