

Module 8.2: Application: Promissory Notes

How do all of those "don't pay until . . ." plans work? The retail industry overflows with financing plans specifically designed to attract customers to purchase merchandise on credit. Most of these offers include terms such as "no money down" and "no payments for x months." You can find these plans at most furniture retailers such as The Brick and Leon's, electronic retailers such as Best Buy, along with many other establishments including jewellery stores and sleep centres.

But have you ever considered how this works on the business side? If every customer purchased merchandise under these no-money-required plans, how would the retailer stay in business? For example, perhaps The Brick's customers purchase furniture in January 2014 that they do not have to pay for until January 2016. During these two years, The Brick does not get paid for the products sold; however, it has paid its suppliers for the merchandise. How can a retailer stay in business while it waits for all those postponed payments?

Consumers generally do not read the fine print on their contracts with these retailers. Few consumers realize that the retailers often sell these contracts (sometimes immediately) to finance companies they have partnered with. While the consumer sees no noticeable difference, behind the scenes the retailer receives cash today in exchange for the right to collect payment in the future when the contract becomes due. That way the finance company becomes responsible for collecting on the loan to the consumer.

This section introduces the mathematics behind the sale of promissory notes between companies. If the debt allows for interest to accumulate, then it is called an *interest-bearing promissory note*. If there is no allowance for interest, then it is called a *noninterest-bearing promissory note*. When promissory notes extend more than one year, they involve compound interest instead of simple interest.

What Are Promissory Notes?

Promissory notes are borrowing tools that allow both companies and people to get financing from a source other than a bank. The money is borrowed from other people or companies who are willing to provide the financing. The Bills of Exchange Act defines a **promissory note** as "an unconditional promise in writing made by one person to another person, signed by the maker, engaging to pay, on demand or at a fixed or determinable future time, a sum certain in money to, or to the order of, a specified person or to bearer."¹ Promissory notes are commonly referred to just as *notes*.

A promissory note is not the same as an IOU (I owe you). An IOU simply acknowledges a debt but contains no promise of repayment, no timelines, and no consequences. A promissory note includes these elements.

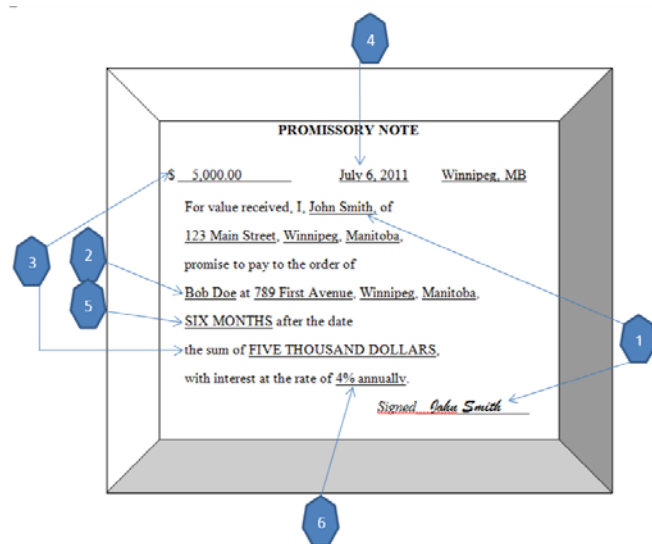
In the opening scenario, it is probably best to create a promissory note that clearly details your agreement and terms.

This allows both of you to have peace of mind in the transaction, and in the event something goes wrong both of you have a

legally binding written record of your transaction.

The illustration here provides an example of a promissory note. Observe the following characteristics:

1. **Borrower.** This is the individual who is borrowing the money, sometimes known as the maker, promisor, or payor. It is best to provide details such as the name and address of the borrower to clearly establish an identity.
2. **Lender.** This is the individual who is loaning the money (and should be repaid), sometimes known as the payee or issuer. As with the borrower, detailed name and address information clearly establish the identity of this person.
3. **Face Value.** This is the borrowed principal. It appears numerically and in writing to prevent it from being altered. In simple interest calculations, this is the P.
4. **Issue Date (Start Date).** This is the date on which the promissory note is issued and forms the starting date of the transaction.



5. **Term.** This is the length of time for which the face value is borrowed from the lender.

6. **Interest Rate.** This is the percentage interest being charged along with the respective time frame for the interest (monthly, annually, etc.). In simple interest calculations, this is the rate (r).

Note in the illustration that the lender has decided to not have any additional consequence for failing to pay back the \$5,000 after the six months. As written, any late payment continues to be charged 4% annual interest only.

Promissory notes are issued outside the normal banking system, so it is up to the lender to determine the interest rate. This results in two types of promissory notes:

1. **Noninterest-Bearing Notes.** In these cases, the lender has decided not to charge any interest, hence making the interest rate 0%. Alternatively, the lender and borrower may decide to include an amount of interest in the face value specified on the note. For example, the lender may borrow \$5,000, but the note is written for \$5,100 with no interest. In the case of a noninterest-bearing note, of course, no calculation of interest on the promissory note is needed to determine the amount that is due, as the face value equals the maturity value.
2. **Interest-Bearing Notes.** In these cases, the lender has decided to charge interest on the face value. This requires you to calculate the interest on the face value. Therefore, the maturity value is the face value plus an interest amount. The method for this calculation is discussed next.

Interest-Bearing Promissory Notes

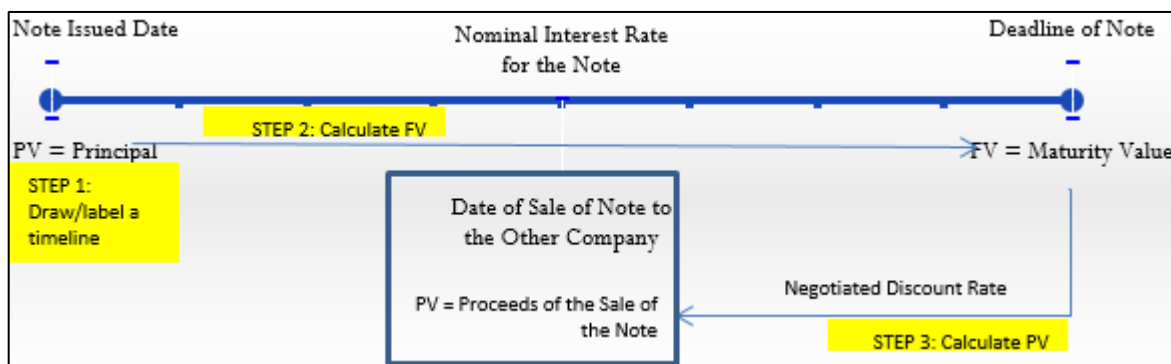
When you participate in "don't pay until . . ." promotions, you create a promissory note in which you promise to pay for your goods within the time interval stated. These promotions commonly carry 0% interest if paid before the stated deadline, so the notes are noninterest-bearing promissory notes. However, failure to pay the note before the deadline transforms the note into an interest-bearing note for which interest is retroactive to the date of sale, usually at a very high rate of interest such as 21%.

The mathematics of interest-bearing promissory notes deal primarily with the sale of long-term promissory notes between organizations. When the note is sold, the company buying it (usually a finance company) purchases the maturity value of the note and not the principal of the note. To the finance company, the transaction is an investment from which it intends to earn a profit through the difference between maturity value and purchase price. Thus, the finance company discounts the maturity value of the note using a discount rate that permits it to invest a smaller sum of money today to receive a larger sum of money in the future. The company selling the note is willing to take the smaller sum of money to cash in its accounts receivables and eliminate the risk of default on the debt.



How It Works

Step 1: Draw a timeline, similar to the one on the next page, detailing the original promissory note and the sale of the note.



Step 2: Take the initial principal on the date of issue and determine the note's future value at the stated deadline using the stated rate of interest attached to the note. As most long-term promissory notes have a fixed rate of interest, this involves a future value calculation using Formula 4.1.6.

- a. Calculate the periodic interest rate using Formula 4.1.4, $i = \frac{IY}{CY}$
- b. Calculate the number of compounding periods between the issue date and due date using Formula 4.1.5, $N = CY \times \text{Years}$.
- c. Solve for the future value using Formula 4.1.6, $FV = PV(1 + i)^N$.

Step 3: Using the date of sale, discount the maturity value of the note using a new negotiated discount rate of interest to determine the proceeds of the sale. Most commonly the negotiated discount rate is a fixed rate and involves a present value calculation using Formula 4.1.6.

- Calculate the new periodic interest rate using Formula 4.1.4, $i = \frac{IY}{CY}$
- Calculate the number of compounding periods between the date of sale of the note and the due date using Formula 4.1.5, $N = CY \times \text{Years}$. Remember to use the CY for the discount rate, not the CY for the original interest rate.
- Solve for the present value using Formula 4.1.6, $FV = PV(1 + i)^N$, rearranging for PV.

Assume that a three-year \$5,000 promissory note with 9% compounded monthly interest is sold to a finance company 18 months before the due date at a discount rate of 16% compounded quarterly.

Step 1: The timeline to the right illustrates the situation.

Step 2a: The periodic interest rate on the note is $i = 9\%/12 = 0.75\%$.

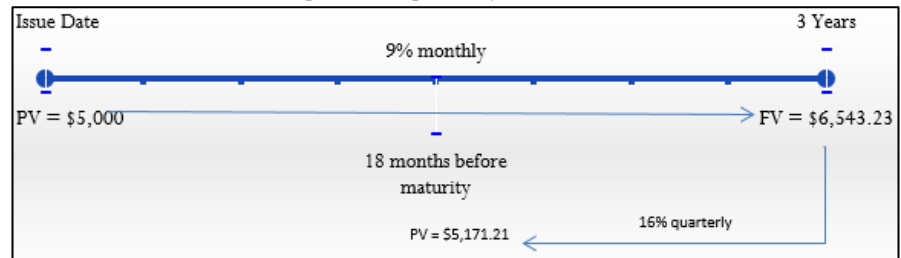
Step 2b: The term is three years with monthly compounding, resulting in $N = 12 \times 3 = 36$.

Step 2c: The maturity value of the note is $FV = \$5,000(1 + 0.0075)^{36} = \$6,543.23$.

Step 3a: Now sell the note. The periodic discount rate is $i = 16\%/4 = 4\%$.

Step 3b: The time before the due date is $1\frac{1}{2}$ years at quarterly compounding. The number of compounding periods is $N = 4 \times 1\frac{1}{2} = 6$.

Step 3c: The proceeds of the sale of the note is $\$6,543.23 = PV(1 + 0.04)^6$, where $PV = \$5,171.21$. The finance company purchases the note (invests in the note) for \$5,171.21. Eighteen months later, when the note is paid, it receives \$6,543.23.



Important Notes

The assumption behind the three-step procedure for selling a long-term promissory note is that the process starts with the issuance of the note and ends with the proceeds of the sale. However, mathematically you may deal with any part of the transaction as an unknown. For example, perhaps the details of the original note are known, the finance company's offer on the date of sale is known, but the quarterly discounted rate used by the finance company needs to be calculated.

The best strategy in any of these scenarios is always to execute step 1 and create a timeline. Identify the known variables to visualize the process, then recognize any variable(s) remaining unknown. Keeping in mind how the selling of a promissory note works, you can adapt the three-step promissory note procedure using any of the techniques discussed in Module 4. Some examples of these adaptations include the following:

- The discounted rate is unknown. Execute steps 1 and 2 normally. In step 3, solve for i (then IY) instead of PV.
- The original principal of the note is unknown. Execute step 1 normally. Work with step 3, but solve for FV instead of PV. Then work with step 2 and solve for PV instead of FV.
- The length of time by which the date of sale precedes the maturity date is unknown. Execute steps 1 and 2 normally. In step 3, solve for N instead of PV.

As you can see, the three steps always stay intact. However, you may need to reverse steps 2 and 3 or calculate a different unknown variable.



Things To Watch Out For

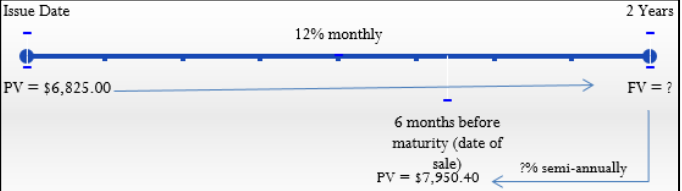
In working with compound interest long-term promissory notes, the most common mistakes relate to the maturity value and the two interest rates.

1. **Maturity Value.** Remember that the company purchasing the note is purchasing the maturity value of the note, *not* its principal on the issue date. Any promissory note situation always involves the maturity value of the promissory note on its due date.
2. **Two Interest Rates.** The sale involves two interest rates: an interest rate tied to the note itself and an interest rate (the discount rate) used by the purchasing company to acquire the note. Do not confuse these two rates.

Example 5.2A: Proceeds on an Interest-Bearing Note		
<p>Jake's Fine Jewellers sold a diamond engagement ring to a customer for \$4,479.95 and established a promissory note under one of its promotions on January 1, 2014. The note requires 6% compounded semi-annually interest and is due on January 1, 2017. On October 1, 2015, Jake's Fine Jewellers needed the money and sold the note to a finance company at a discount rate of 11% compounded quarterly. What are the proceeds of the sale?</p>		
Plan	Find the proceeds of the sale for Jake's Fine Jewellers on October 1, 2013. This is the present value of the note (PV) based on the maturity value and the discount rate.	
Understand	<p>What You Already Know</p> <p>Step 1: The issue date, maturity date, principal, interest rate on the note, date of sale, and the discount rate are known, as illustrated in the timeline.</p>	<p>How You Will Get There</p> <p>Step 2a: Working with the promissory note itself, calculate the periodic interest rate by applying Formula 4.1.4.</p> <p>Step 2b: Calculate the number of compound periods using Formula 4.1.5.</p> <p>Step 2c: Calculate the maturity value of the note using Formula 4.1.6.</p> <p>Step 3a: Working with the sale of the note, calculate the discount periodic interest rate by applying Formula 4.1.4.</p> <p>Step 3b: Calculate the number of compound periods elapsing between the sale and maturity. Use Formula 4.1.5.</p>
Perform	<p>Step 2a: $IY = 6\%$, $CY = 2$, $i = 6\%/2 = 3\%$</p> <p>Step 2b: Years = January 1, 2017 – January 1, 2014 = 3 Years, $N = 2 \times 3 = 6$</p> <p>Step 2c: $PV = \\$4,479.95$, $FV = \\$4,479.95(1 + 0.03)^6 = \\$5,349.29$</p> <p>Step 3a: $IY = 11\%$, $CY = 4$, $i = 11\%/4 = 2.75\%$</p> <p>Step 3b: Years = January 1, 2017 – October 1, 2015 = $1\frac{1}{4}$ Years, $N = 4 \times 1\frac{1}{4} = 5$</p> <p>Step 3c: $\\$5,349.29 = PV(1 + 0.0275)^5$ $PV = \frac{\\$5,349.29}{1.0275^5} = \\$4,670.75$</p>	
Present	<p>Jake's Fine Jewellers made the sale for \$4,479.95 on January 1, 2014. On October 1, 2015, it receives \$4,670.75 in proceeds of the sale to the finance company. The finance company holds the note until maturity and receives \$5,349.29 from the customer.</p>	

Example 5.2B: Finding an Unknown Discount Rate

A \$6,825 two-year promissory note bearing interest of 12% compounded monthly is sold six months before maturity to a finance company for proceeds of \$7,950.40. What semi-annually compounded discount rate was used by the finance company?

Plan	Calculate the semi-annually compounded negotiated discount rate (IY) used by the finance company when it purchased the note.	
Understand	<p>What You Already Know</p> <p>Step 1: The term, principal, promissory note interest rate, date of sale, and proceeds amount are known, as shown in the timeline.</p>	<p>How You Will Get There</p> <p>Step 2a: Working with the promissory note itself, calculate the periodic interest rate by applying Formula 4.1.4.</p> <p>Step 2b: Calculate the number of compound periods using Formula 4.1.5.</p> <p>Step 2c: Calculate the maturity value of the note using Formula 4.1.6.</p> <p>Step 3a: Note the CY on the discount rate.</p> <p>Step 3b: Calculate the number of compound periods elapsing between the sale and before maturity. Use Formula 4.1.5.</p> <p>Step 3c: Calculate the periodic discount rate used by the finance company by applying Formula 4.1.5 and rearranging for i. Then substitute into Formula 4.1.4 and rearrange for IY.</p> 
Perform	<p>Step 2a: $IY = 12\%$, $CY = 12$, $i = 12\%/12 = 1\%$</p> <p>Step 2b: $Years = 2$, $N = 12 \times 2 = 24$</p> <p>Step 2c: $PV = \\$6,825.00$, $FV = \\$6,825.00(1 + 0.01)^{24} = \\$8,665.94$</p> <p>Step 3a: $CY = 2$</p> <p>Step 3b: $Years = 6 \text{ months} = \frac{1}{2} \text{ Year}$, $N = 2 \times \frac{1}{2} = 1$</p> <p>Step 3c: Solve for i:</p> $\$8,665.94 = \$7,950.40(1 + i)^1$ $1.090000 = 1 + i$ $i = 0.090000$ <p>Solve for IY: $0.090000 = IY \div 2$</p> <p>$IY = 0.180001$ or 18.0001% compounded semi-annually (most likely 18%; the difference is due to a rounding error)</p>	
Present	The sale of the promissory note is based on a maturity value of \$8,665.94. The finance company used a discount rate of 18% compounded semi-annually to arrive at proceeds of \$7,950.40.	

Noninterest-Bearing Promissory Notes

A noninterest-bearing promissory note involves either truly having 0% interest or else already including a flat fee or rate within the note's face value. Therefore, the principal amount and maturity amount of the promissory note are the same.



How It Works

A noninterest-bearing note simplifies the calculations involved with promissory notes. Instead of performing a future value calculation on the principal in step 2, your new step 2 involves equating the present value and maturity value to the same amount ($PV = FV$). You then proceed with step 3.

Assume a three-year \$5,000 noninterest-bearing promissory note is sold to a finance company 18 months before the due date at a discount rate of 16% compounded quarterly.

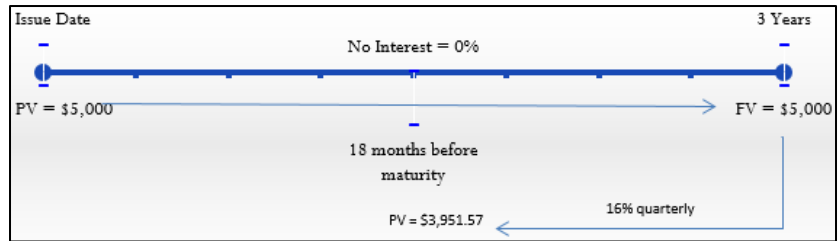
Step 1: The timeline is illustrated here.

Step 2: The maturity value of the note three years from now is the same as the principal, or $FV = \$5,000$.

Step 3a: Now sell the note. The periodic discount rate is $i = 16\%/4 = 4\%$.

Step 3b: The time before the due date is $1\frac{1}{2}$ years at quarterly compounding. The number of compounding periods is $N = 4 \times 1\frac{1}{2} = 6$.

Step 3c: The proceeds of the sale of the note is $\$5,000 = PV(1 + 0.04)^6$, or $PV = \$3,951.57$. The finance company purchases the note (invests in the note) for \$3,951.57. Eighteen months later, when the note is paid, it receives \$5,000.



Example 5.2C: Selling a Noninterest-Bearing Long-Term Promissory Note

A five-year, noninterest-bearing promissory note for \$8,000 was issued on June 23, 2011. The plan is to sell the note at a discounted rate of 4.5% compounded semi-annually on December 23, 2015. Calculate the expected proceeds on the note.

Plan	Calculate the proceeds on the note (PV) based on the note's maturity value and the date of the sale.	
Understand	<p>What You Already Know</p> <p>Step 1: The principal, noninterest on the note, issue date, due date, discount rate, and date of sale are known, as illustrated in the timeline.</p>	<p>How You Will Get There</p> <p>Step 2: Equate the FV of the note with the principal of the note.</p> <p>Step 3a: Working with the sale of the note, calculate the discount periodic interest rate by applying Formula 4.1.4.</p> <p>Step 3b: Calculate the number of compound periods elapsing between the sale and maturity. Use Formula 4.1.5.</p> <p>Step 3c: Calculate the proceeds of the sale by using Formula 4.1.6, rearranging for PV.</p>
Perform	<p>Step 2: $FV = PV = \\$8,000$</p> <p>Step 3a: $IY = 4.5\%$, $CY = 2$, $i = 4.5\%/2 = 2.25\%$</p> <p>Step 3b: $\text{Years} = \text{June 23, 2017} - \text{December 23, 2015}$ $= 1\frac{1}{2} \text{ Years}$, $N = 2 \times 1\frac{1}{2} = 3$</p> <p>Step 3c: $\\$8,000 = PV(1 + 0.0225)^3$ $PV = \frac{\\$8,000}{1.0225^3} = \\$7,483.42$</p>	
Present	The expected proceeds are \$7,483.42 on December 23, 2015, with a maturity value of \$8,000.00 on June 23, 2017.	



Module 5.2 Exercises

Mechanics

1. Examine this promissory note.
 - a. Identify the six components of the promissory note.
 - b. Determine the legal due date.
 - c. Calculate the maturity value of the note.

PROMISSORY NOTE		
\$13,500.00	January 7, 2010	Toronto, ON
For value received, I, Joan d'Arc, of		
456 123 rd Street, Toronto, Ontario		
promise to pay to the order of		
Candlelight Industries at 99 Broad Ave., Toronto, Ontario		
NINE MONTHS after the date		
the sum of THIRTEEN THOUSAND AND FIVE		
HUNDRED DOLLARS,		
with interest at the rate of 9% annually.		
Signed, <u>Joan d'Arc</u>		

Calculate the unknown variable (indicated with a ?) for the following noninterest-bearing promissory notes.

	Principal	Issue Date	Due Date	Sale Date	Discount Rate	Sale Proceeds
2.	\$10,000	August 14, 2010	November 14, 2015	February 14, 2012	5.95% compounded quarterly	\$?
3.	\$19,000	May 29, 2010	May 29, 2015	September 29, 2012	8.7% compounded monthly	\$?
4.	\$?	September 30, 2009	September 30, 2014	March 30, 2012	6.8% compounded semi-annually	\$21,574.34
5.	\$31,300	June 3, 2009	June 3, 2015	November 3, 2012	?% compounded monthly	\$27,268.08

Calculate the unknown variable(s) (indicated with a ?) for the following interest-bearing promissory notes.

	Issue Amount	Term of Note	Interest Rate on Note	Date of Sale (before maturity)	Discount Rate	Sale Proceeds
6.	\$51,000	6 years	7.75% compounded quarterly	2½ years	12% compounded semi-annually	\$?
7.	\$18,200	4½ years	9.45% compounded monthly	1 year, 6 months	15% compounded quarterly	\$?
8.	\$5,350	3¼ years	6.95% compounded monthly	1½ years	?% compounded semi-annually	\$5,587.02
9.	\$2,900	4 years	8.8% compounded semi-annually	9 months	?% compounded monthly	\$3,570.13
10.	\$?	5 years	7.5% compounded quarterly	2 years, 6 months	13.25% compounded semi-annually	\$11,705.14
11.	\$26,945.75	6 years	10.3% compounded annually	? years, ? months	19.3% compounded monthly	\$32,046.52

Applications

12. Determine the proceeds of the sale on a seven-year noninterest-bearing promissory note for \$1,600, discounted 45 months before its due date at a discount rate of 8.2% compounded quarterly.
13. Determine the proceeds of the sale on a six-year interest-bearing promissory note for \$5,750 at 6.9% compounded monthly, discounted two years and three months before its due date at a discount rate of 9.9% compounded quarterly.
14. A three-year interest-bearing promissory note for \$8,900 at 3.8% compounded annually is sold one year and two months before its due date at a discount rate of 7.1% compounded monthly. What is the amount of the discount on the sale?
15. Two years and 10 months before its due date, an eight-year interest-bearing promissory note for \$3,875 at 2.9% compounded semi-annually is discounted to have sale proceeds of \$4,182.10. What monthly compounded discount rate was used?
16. On May 1, 2012, a six-year interest-bearing note for \$8,800 at 4.99% compounded monthly dated August 1, 2009, is discounted at 8% compounded semi-annually. Determine the proceeds on the note.
17. A seven-year interest-bearing note for \$19,950 at 8.1% compounded quarterly is issued on January 19, 2006. Four years and 11 months later, the note is discounted at 14.55% compounded monthly. Determine the proceeds on the note and how much interest the original owner of the note realized.

Challenge, Critical Thinking, & Other Applications

18. A \$36,555 interest-bearing note at 5% compounded monthly is issued on October 15, 2011, for a term of 87 months. Fifty-seven months later, the note is sold to yield a discount amount of \$11,733.41. What quarterly compounded discount rate is being used?
19. On December 12, 2012, an eight-year promissory note at 6.2% compounded semi-annually with three years and three months remaining until its due date is sold. The discount rate is 10.9% compounded monthly, resulting in a discount of \$49,349.87. Calculate the original principal of the note.
20. A 10-year, \$100,000 note at 7.5% compounded quarterly is to be sold. The prevailing discount rate for promissory notes of this type today, six years before maturity, is 9% compounded annually. Prevailing discount rates are forecast to rise by 2% every year. The seller of the note will sell the note today, one year from today, or two years from today, depending on which sale date produces the highest nominal proceeds. Rank the various alternatives and recommend a sale date.