Module 11.2 - Venn Diagrams and Cardinality

Important Topics of this Section

Venn Diagrams
Visualizing the union and intersection of sets using Venn Diagrams
Cardinality of a set
Cardinality properties
Finding cardinality using a Venn diagram

To visualize the interaction of sets, John Venn in 1880 thought to use overlapping circles, building on a similar idea used by Leonhard Euler in the 18th century. These illustrations now called **Venn Diagrams**.

Venn Diagram

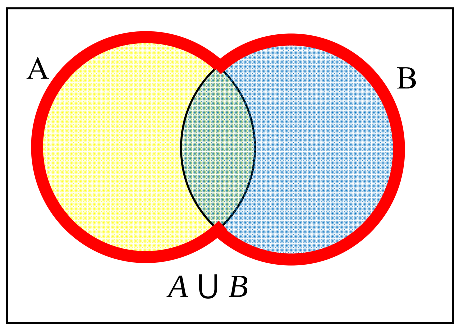
A Venn diagram represents each set by a circle, usually drawn inside of a containing box representing the universal set. Overlapping areas indicate elements common to both sets.

Basic Venn diagrams can illustrate the interaction of two or three sets.

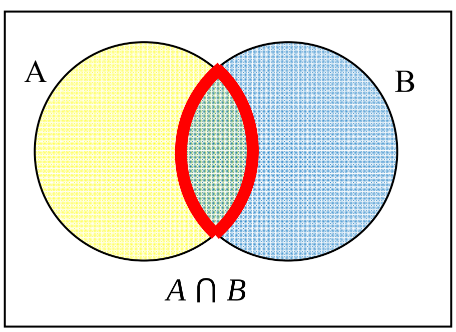
Example 1

Create Venn diagrams to illustrate $A \cup B$, $A \cap B$, and $A^c \cap B$

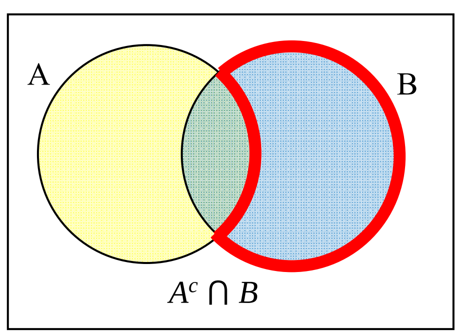
$A \cup B$ contains all elements in *either* set.



$A \cap B$ contains only those elements in both sets – in the overlap of the circles.



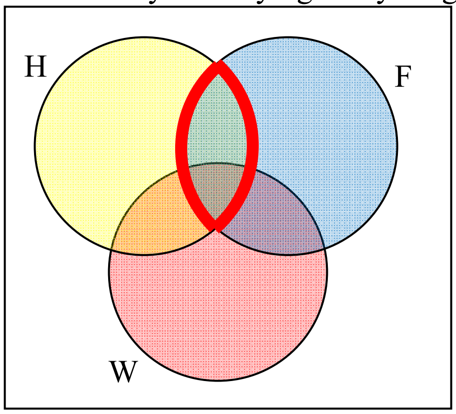
A^c will contain all elements *not* in the set A. $A^c \cap B$ will contain the elements in set B that are not in set A.



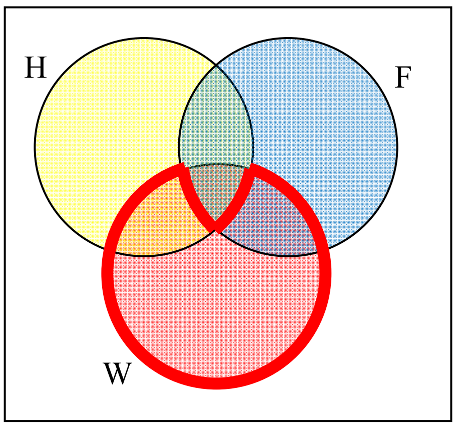
Example 2

Use a Venn diagram to illustrate $(H \cap F)^c \cap W$

We'll start by identifying everything in the set $H \cap F$

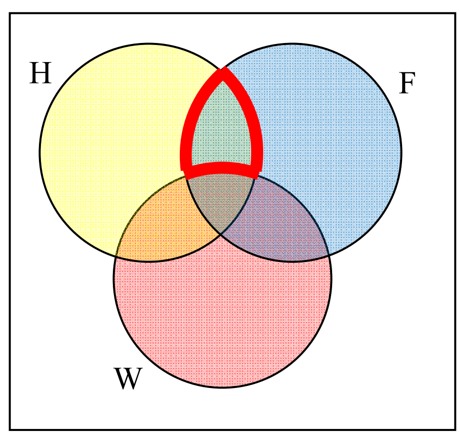


Now, $(H \cap F)^c \cap W$ will contain everything *not* in the set identified above that is also in set *W*.



Example 3

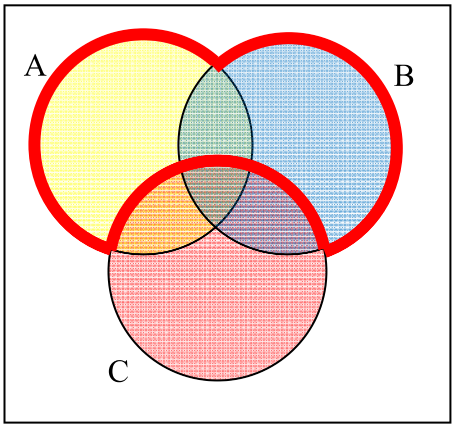
Create an expression to represent the outlined part of the Venn diagram shown.



The elements in the outlined set *are* in sets *H* and *F*, but are not in set *W*. So we could represent this set as $H \cap F \cap W^c$

Try it Now

1. Create an expression to represent the outlined portion of the Venn diagram shown



Cardinality

Often times we are interested in the number of items in a set or subset. This is called the cardinality of the set.

Cardinality

The number of elements in a set is the cardinality of that set.

The cardinality of the set A is often notated as $|A|$ or $n(A)$

Example 4

Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8\}$.
What is the cardinality of B? $A \cup B$, $A \cap B$?

The cardinality of B is 4, since there are 4 elements in the set.
The cardinality of $A \cup B$ is 7, since $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$, which contains 7 elements.
The cardinality of $A \cap B$ is 3, since $A \cap B = \{2, 4, 6\}$, which contains 3 elements.

Example 5

What is the cardinality of P = the set of English names for the months of the year?

The cardinality of this set is 12, since there are 12 months in the year.

Sometimes we may be interested in the cardinality of the union or intersection of sets, but not know the actual elements of each set. This is common in surveying.

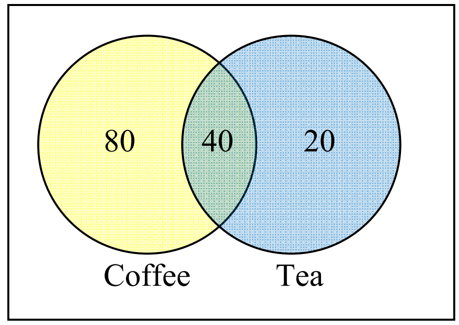
Example 6

A survey asks 200 people "What beverage do you drink in the morning", and offers choices:

- Tea only
- Coffee only
- Both coffee and tea

Suppose 20 report tea only, 80 report coffee only, 40 report both. How many people drink tea in the morning? How many people drink neither tea or coffee?

This question can most easily be answered by creating a Venn diagram. We can see that we can find the people who drink tea by adding those who drink only tea to those who drink both: 60 people.



We can also see that those who drink neither are those not contained in the any of the three other groupings, so we can count those by subtracting from the cardinality of the universal set, 200.
200 – 20 – 80 – 40 = 60 people who drink neither.

Example 7

A survey asks: Which online services have you used in the last month:

- Twitter
- Facebook
- Have used both

The results show 40% of those surveyed have used Twitter, 70% have used Facebook, and 20% have used both. How many people have used neither Twitter or Facebook?

Let T be the set of all people who have used Twitter, and F be the set of all people who have used Facebook. Notice that while the cardinality of F is 70% and the cardinality of T is 40%, the cardinality of $F \cup T$ is not simply 70% + 40%, since that would count those who use both services twice. To find the cardinality of $F \cup T$, we can add the cardinality of F and the cardinality of T, then subtract those in intersection that we've counted twice.

In symbols,
$n(F \cup T) = n(F) + n(T) - n(F \cap T)$
$n(F \cup T) = 70\% + 40\% - 20\% = 90\%$

Now, to find how many people have not used either service, we're looking for the cardinality of $(F \cup T)^c$. Since the universal set contains 100% of people and the cardinality of $F \cup T$ = 90%, the cardinality of $(F \cup T)^c$ must be the other 10%.

The previous example illustrated two important properties

Cardinality properties

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A^c) = n(U) - n(A)$

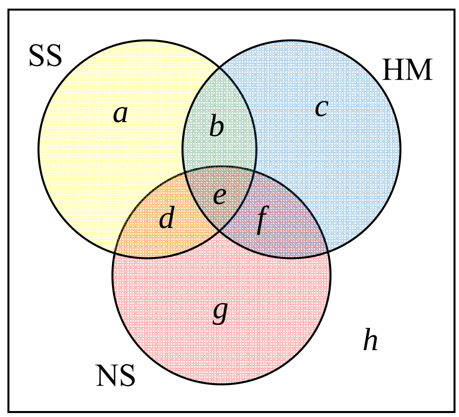
Notice that the first property can also be written in an equivalent form by solving for the cardinality of the intersection:
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

Example 8

Fifty students were surveyed, and asked if they were taking a social science (SS), humanities (HM) or a natural science (NS) course the next quarter.

21 were taking a SS course	26 were taking a HM course
19 were taking a NS course	9 were taking SS and HM
7 were taking SS and NS	10 were taking HM and NS
3 were taking all three	7 were taking none

How many students are only taking a SS course?



It might help to look at a Venn diagram.
From the given data, we know that there are 3 students in region *e* and 7 students in region *h*.

Since 7 students were taking a SS and NS course, we know that n(*d*) + n(*e*) = 7. Since we know there are 3 students in region 3, there must be 7 – 3 = 4 students in region *d*.

Similarly, since there are 10 students taking HM and NS, which includes regions *e* and *f*, there must be 10 – 3 = 7 students in region *f*.

Since 9 students were taking SS and HM, there must be 9 – 3 = 6 students in region *b*.

Now, we know that 21 students were taking a SS course. This includes students from regions *a*, *b*, *d*, and *e*. Since we know the number of students in all but region *a*, we can determine that 21 – 6 – 4 – 3 = 8 students are in region *a*.

8 students are taking only a SS course.

Try it Now

2. One hundred fifty people were surveyed and asked if they believed in UFOs, ghosts, and Bigfoot.

43 believed in UFOs
44 believed in ghosts
25 believed in Bigfoot
10 believed in UFOs and ghosts
8 believed in ghosts and Bigfoot
5 believed in UFOs and Bigfoot
2 believed in all three

How many people surveyed believed in at least one of these things?

Try it Now Answers

1. $A \cup B \cap C^c$

2. Starting with the intersection of all three circles, we work our way out. Since 10 people believe in UFOs and Ghosts, and 2 believe in all three, that leaves 8 that believe in only UFOs and Ghosts. We work our way out, filling in all the regions. Once we have, we can add up all those regions, getting 91 people in the union of all three sets. This leaves 150 – 91 = 59 who believe in none.