MWF 12, F19

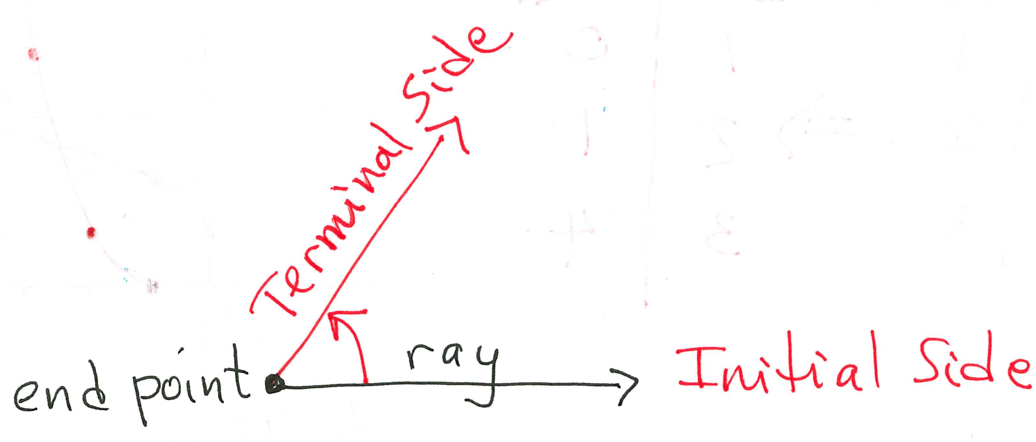
Chapter 2. Trigonometric Functions

2.1 Angles and Their Measure

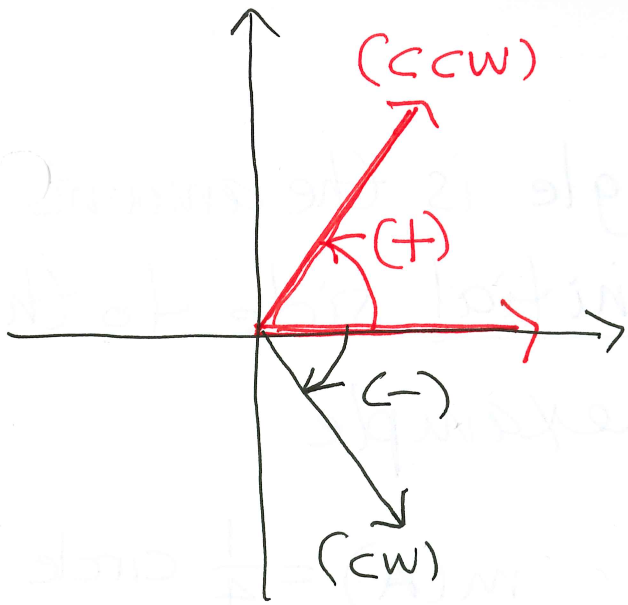
Trigonometry – derived from the Greek words meaning "measure of triangles".
Today, trig. is used to describe many natural phenomena such vibrations, sound waves, light waves, electricity, and functions in fields such biology, economics, surveying, etc.

- Angles

From Geometry, an angle is determined by rotating a ray about its end point. TO BE CONT'D



We can place such an angle on the xy-plane with its vertex at the origin and the initial side on the (+) x-axis.



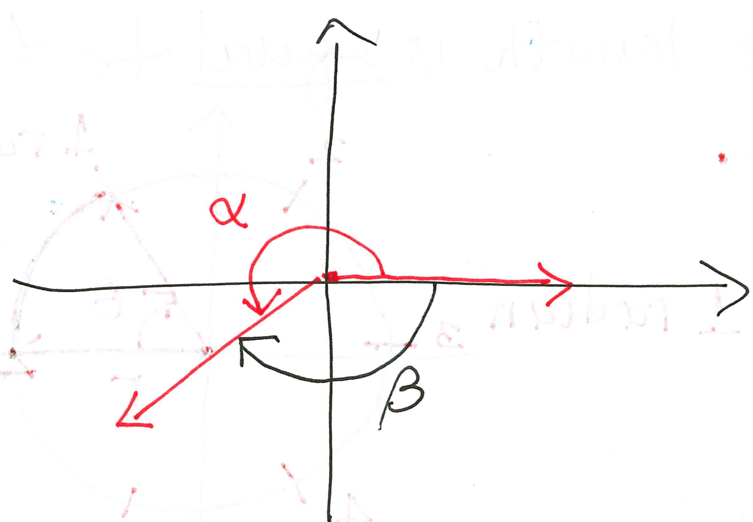
The measure of this angle is <u>positive</u> if the rotation is counter clockwise (ccw)

The measure will be <u>negative</u> if the rotation is clockwise (cw)

Angles can be "named" with:

- Greek letters: $\alpha, \beta, \theta, \Phi, \omega$, etc.
- Capital English Letters: A, B, ...
- Variables: x, y, z, etc.

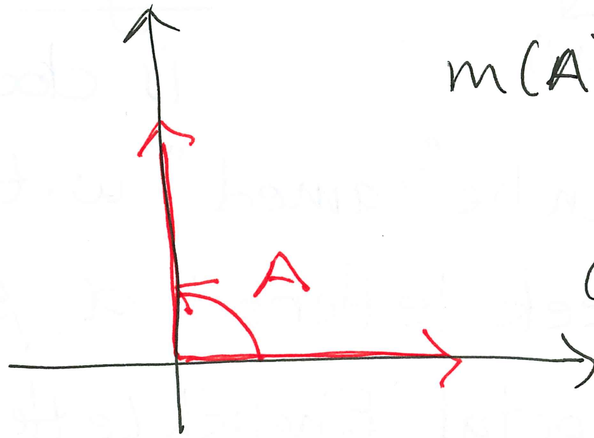
<u>Def</u> - angles with the <u>same</u> <u>initial</u> and <u>terminal</u> sides are called <u>coterminal</u> angles.



α and β are <u>coterminal</u> angles.

• Radian Measure

The measure of an angle is the amount of rotation from its initial side to the terminal side. For example



$m(A) = \frac{1}{4}$ circle

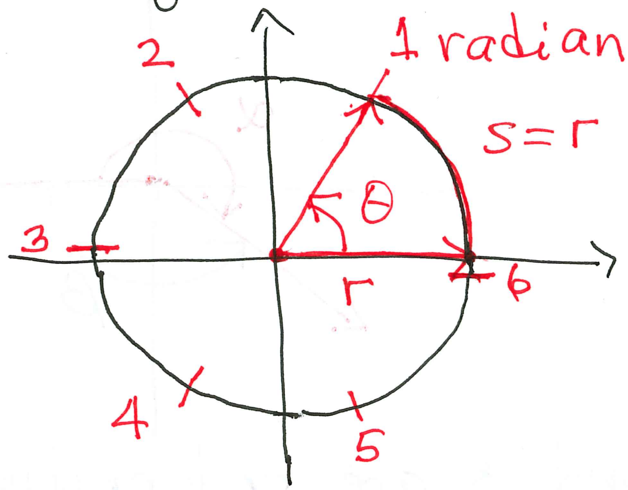
$= 90^\circ$

(right angle)

We can also measure angle in radians, which are nondimensional measures.

Def – one radian is the measure of a central angle θ that intersects (or subtends) an arc s whose length is equal to the radius of the circle.

$\theta = \frac{s}{r} = 1$ radian



For a circle, its circumference $C = d\pi$
$= 2r\pi$

So, if $r = 1$, $C = 2\pi$.

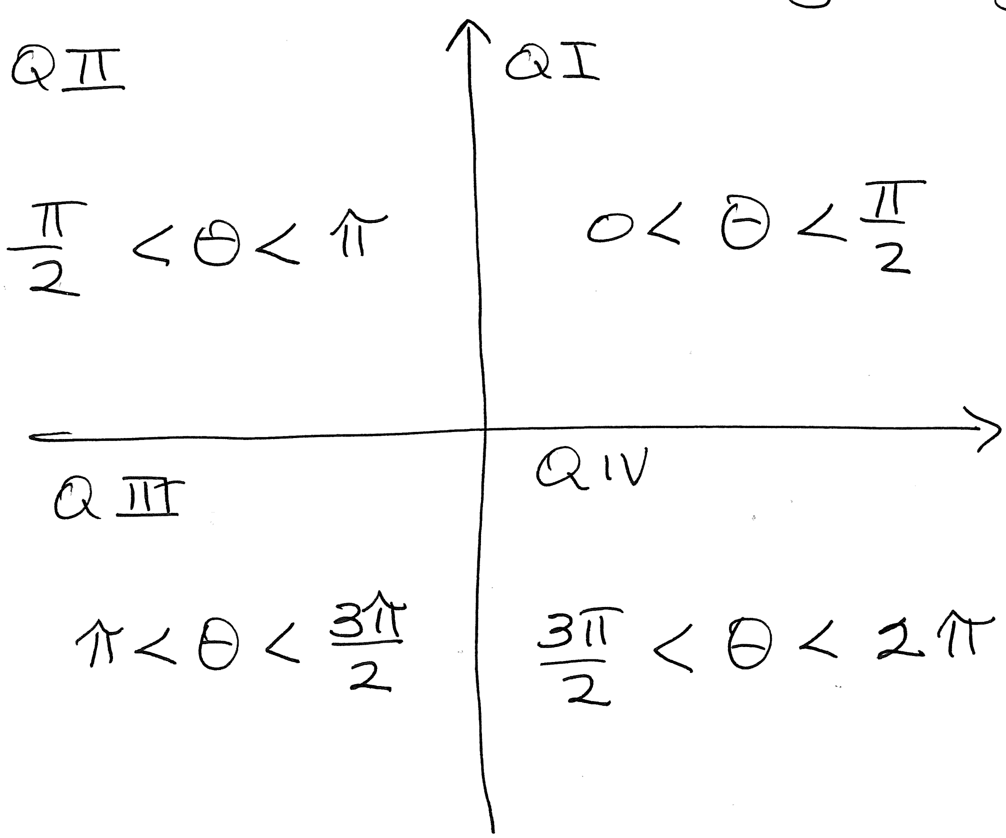
Thus, we can say that an angle that measures 2π radians has the same measure of <u>one full revolution of a circle</u>.

$\underline{1}$ revolution $= 2\pi$ radians

$\frac{1}{2}$ " $= \pi$ radians

$\frac{1}{4}$ " $= \frac{\pi}{2}$ radians

(right angle)



- Degree Measure

We define a degree to be $\frac{1}{360}$ of a revolution.

$$1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ$$
$$\tfrac{1}{2} \quad " \quad = \pi \quad " \quad = 180^\circ$$
$$\tfrac{1}{4} \quad " \quad = \frac{\pi}{2} \quad " \quad = 90^\circ$$

The unit conversion factors for changing from degrees to radians and vice-versa are:

$$\frac{180^\circ}{\pi \text{ radians}} = 1 = \frac{\pi \text{ radians}}{180^\circ}$$

EX: (a) Convert 135° to radians.

$$\overset{3}{\cancel{135}}{}^\circ\left(\frac{\pi \text{ radians}}{\underset{4}{\cancel{180}}{}^\circ}\right) = \underbrace{\frac{3\pi}{4} \text{ radians}}$$

(b) Convert $\frac{2\pi}{3}$ radians to degrees.

$$\left(\frac{2\cancel{\pi}}{\cancel{3}} \cancel{\text{radians}}\right)\left(\frac{\overset{60^\circ}{\cancel{180}}{}^\circ}{\cancel{\pi}\ \cancel{\text{radians}}}\right) = \underbrace{120^\circ}$$

- Length of an Arc of a circle

Def - for a circle of radius r, a central angle of θ radians subtends an arc s whose length is $\boxed{s = r\theta}$.

EX: Find the length of arc s for an angle $\theta = 125°$ on a circle of radius 5 meters. Round answer to 2 decimals.

$$s = r\theta = (5m)(125°)\left(\frac{\pi}{180°}\right)$$

$$\underbrace{s = 10.91\ m}$$

- Area of a Sector of a Circle

Def - the area A of a sector of a circle of radius r formed by a central angle θ in radians is $\boxed{A = \frac{1}{2} r^2 \theta}$.

EX: Find the area of a sector of a circle of radius 2 ft formed by a central angle of 30°. Round answer to 2 decimals.

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(2ft)^2(30°)\left(\frac{\pi}{180°}\right)$$

$$\underbrace{A = 1.05\ ft^2}$$

- D-M-S Notation

Historically, fractional parts of a degree have been measured in minutes and seconds. TO BE CONT'D.

$$1 \text{ minute} = \frac{1}{60}(1^\circ) = 1'$$

$$1 \text{ second} = \frac{1}{3600}(1^\circ) = 1''$$

So, if we are given an angle measure of $22^\circ 15' 45''$, we can convert to decimal notation as follows:

$$22^\circ + 15' + 45'' = 22^\circ + \left(15' \cdot \frac{1^\circ}{60'}\right) + \left(45'' \cdot \frac{1^\circ}{3600''}\right)$$

$$= 22^\circ + 0.25^\circ + 0.0125^\circ = \underbrace{22.2625^\circ}$$

What if we need to change 40.32° to D-M-S notation?

$$40.32^\circ = 40^\circ + 0.32^\circ\left(\frac{60'}{1^\circ}\right)$$

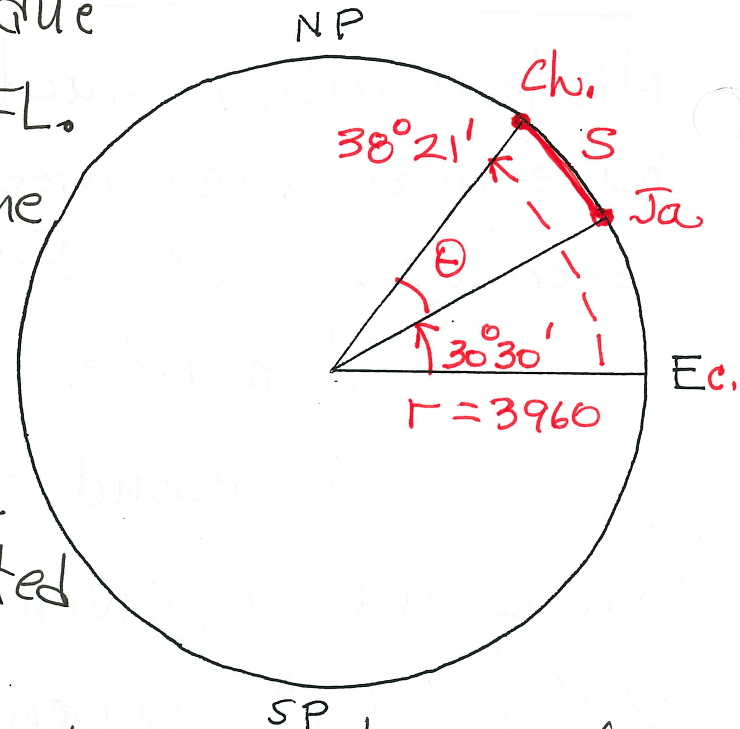
$$= 40^\circ + 19.2'$$

$$= 40^\circ + 19' + 0.2'\left(\frac{60''}{1'}\right)$$

$$= 40^\circ + 19' + 12''$$

$$= \underbrace{40^\circ 19' 12''}$$

EX: Charleston, WV is due north of Jacksonville, FL. Find the distance to the nearest mile between Charleston located at 38° 21' north latitude and Jacksonville located at 30° 30' north latitude. Assume that the radius of the earth is 3,960 miles.



We know that for any circle with a central angle θ, $s = r\theta$, wher θ has to be in radians.

$$\theta^\circ = \begin{array}{r} 38^\circ 21' \\ -30^\circ 30' \\ \hline \end{array} \Rightarrow \begin{array}{r} 37^\circ 81' \\ -30^\circ 30' \\ \hline 7^\circ 51' \end{array} = 7^\circ + \left(\frac{51'}{60'}\right)^\circ = \underline{7.85^\circ = \theta}$$

$$s = r\theta = (3{,}960 \text{ miles})(7.85^\circ)\left(\frac{\pi}{180^\circ}\right)$$

$$\underline{s = 543 \text{ miles}}$$