(b) How long will it take for the thermometer to indicate 39°?

$$39 = 38 + 34e^{-0.21766t}$$

$$1 = 34e^{-0.21766t}$$

$$\frac{1}{34} = e^{-0.21766t}$$

$$\ln\left(\frac{1}{34}\right) = -0.21766t$$

$$\frac{\ln\left(\frac{1}{34}\right)}{-0.21766} = t = 16.2 \text{ minutes}$$

7.6 Continued

- Logistic Models

Nothing grows exponentially forever. A more realistic model that contains exp. growth (or decay) is the logistic model:

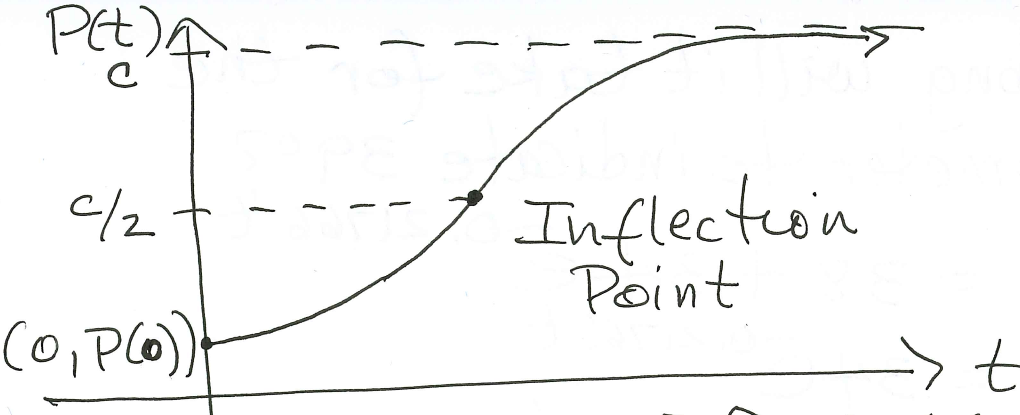
$$P(t) = \frac{c}{1 + ae^{-bt}}$$

where a, b, c are constants with $c > 0$.

If $b > 0$, the function models growth

If $b < 0$, the function models decay.

A typical graph of logistic growth is as follows:



TO BE CONT'D - 7.6 Continued

EX: Fruit flies in container with nutrients have a population P given by the function

$$P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$$

where t = days.

FIND: (a) the initial population

$$P(0) = \frac{230}{1 + 56.5\cancel{e^0}_{1}} = \frac{230}{57.5} = 4 \text{ flies}$$

(b) How long will it take for the population to become 180 flies?

$$180 = \frac{230}{1 + 56.5e^{-0.37t}}$$

$$180(1 + 56.5e^{-0.37t}) = 230$$

$$1 + 56.5e^{-0.37t} = \frac{23}{18}$$

$$56.5e^{-0.37t} = \frac{23}{18} - \frac{18}{18} = \frac{5}{18}$$

$$e^{-0.37t} = \frac{5}{18(56.5)}$$

$$\frac{-0.37t}{-0.37} = \frac{\ln\left(\frac{5}{18(56.5)}\right)}{-0.37}$$

$t = 14.4$ days

EX: The spread of a cold varius through a college campus is given by

$$s(t) = \frac{4000}{1+3900e^{-0.7t}}$$

where s = no. of students who get sick
t = time in days.

FIND: (a) how many students are sick after 5 days?

$$s(5) = \frac{4000}{1+3900e^{-0.7(5)}} = \underline{34 \text{ students}}$$

(b) How long will it take for 1000 students to get sick?

$$1000 = \frac{4000}{1+3900e^{-0.7t}}$$

$$1000(1+3900e^{-0.7t}) = 4000$$

$$1+3900e^{-0.7t} = 4$$

$$3900e^{-0.7t} = 3$$

$$e^{-0.7t} = \frac{3}{3900}$$

$$\frac{-\cancel{0.7}\,t}{-\cancel{0.7}} = \frac{\ln\left(\frac{3}{3900}\right)}{-0.7}$$

$$t = \frac{\ln\left(\frac{3}{3900}\right)}{-0.7} = \underline{10.2 \text{ days}}$$

(c) What is the maximum no. of students who could get sick?

As $t \to \infty$, $e^{-0.7t} \to 0$, $s \Rightarrow \underline{4000 \text{ students}}$.

EX: Suppose a population of bald eagles that was started in Montana has grown to 6 eagles after 1 year, 40 eagles after 10 years, and 200 eagles after 20 years. A logistic model that represents this growth was calculated statistically to be

$$P(t) = \frac{394.5}{1 + 80.8 e^{-0.221 t}}.$$

FIND: (a) the number of eagles after 25 years.

$$P(25) = \frac{394.5}{1 + 80.8 e^{-0.221(25)}} = 298$$

eagle.

(b) What would be maximum expected capicity of this environment for this population of eagles?

$$P(t)_{max} = 394 \text{ eagles}$$

THE END