MWF 12, F19

MAT 1093.004

Pre-Calculus

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1.3 Functions and Their Graphs

Def - a **set** is a collection of objects called **elements**.

Def - a **relation** is a set of **ordered pairs** (x, y). The set of the **first** coordinates (x's) is called the **domain**. The set of the second coordinates (y's) is called the **range**.

TO BE CONT'D

1.3 Continued

Def - a **function** is a set of **ordered pairs**, (x, y). The set of the first coordinates (x's) is called the domain. A function has each element in the **domain** assigned **exactly one** element in the range.

EX: Given $S = \{(1,1), (2,4), (3,9)\}$.

Is S a function? **Yes**

Most relations are given in equation form, for example, $y = x + 2$.

Is y a function of x?

$y = x+2$

$\Rightarrow$ Yes

x	y
-2	0
-1	1
0	2
1	3
2	4
$\vdots$	

Is $y^2 = x$ a function?

x	y
4	2
4	-2

NOT a function

- Function Notation

Functions can be "named" using any letter. For example, $y = x+2$, could be named the function f and would be written as:

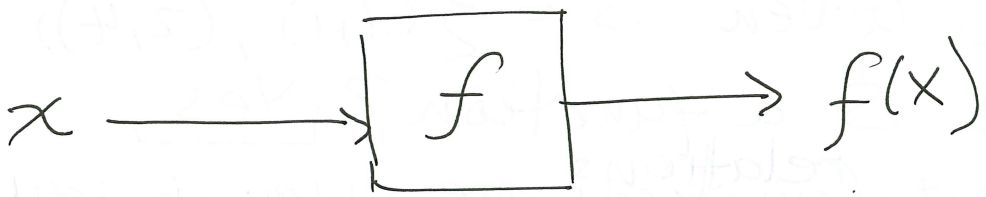
input

$$y = f(x) = x+2$$

output rule

$f(x)$ is read "f of x" or "f at x"

In diagram form:



EX: Given $f(x) = x - 3$.

FIND: (a) $f(7) = 7 - 3 = \underline{4}$ $\Rightarrow (7, 4)$

(b) $f(x+h) = \underline{x + h - 3}$

(c) $$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{x + h - 3 - x + 3}{h} = \underline{1}$$

- Domain of a function - to find all the x-values for which we can calculate a corresponding real y-value.

EX: Find the domain of each function.

(a) $$f(x) = \frac{1}{x^2 - 9}$$

let $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

$d_f = \underline{\{x \mid x \neq -3, x \neq 3\}}$ (set-builder notation)

$= \underline{(-\infty, -3) \cup (-3, 3) \cup (3, \infty)}$ (interval notation)

excluded

(b) $g(x) = 8 + \sqrt{5-x}$ (radical; radicand: $5-x$). Let $5-x \geq 0$

$-x \geq -5$

$x \leq 5$

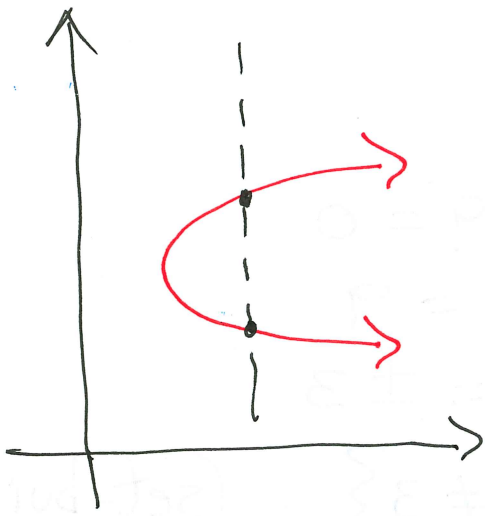
$d_g = \{x \mid x \leq 5\} = (-\infty, 5]$ ← included

- **Identify The Graph of a Function**
  - **Vertical Line Test**

    A relation on the xy-plane is the graph of a function if and only if every vertical line intersects the graph at most at one point.

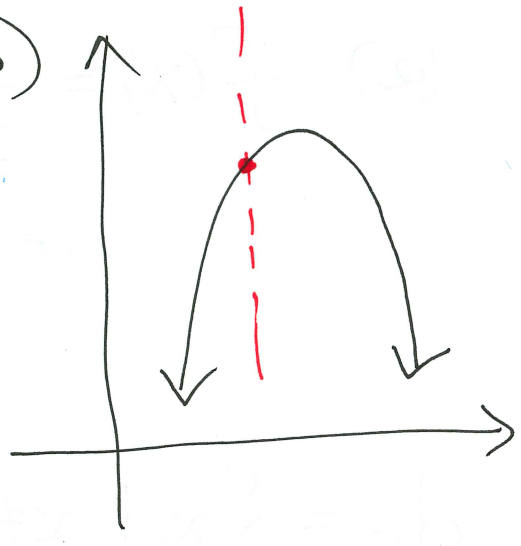
EX: Which of the graphs are functions?

(a)



NOT a function

(b)

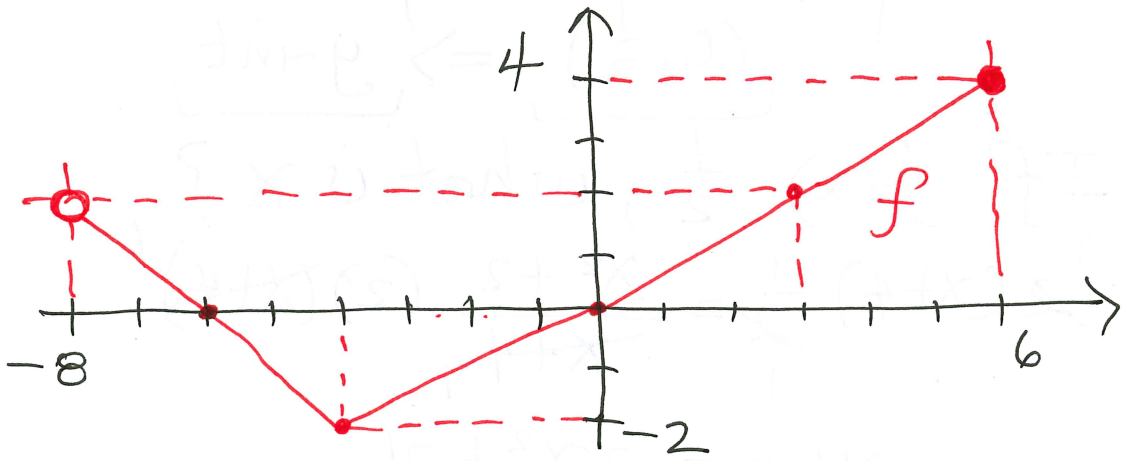


Function

MWF

• Finding Information From Graphs of Functions

EX: Given the graph of f.



FIND:

(a) the domain of f: $(-8, 6] = \{x \mid -8 < x \leq 6\}$

(b) the range of f: $[-2, 4] = \{y \mid -2 \leq y \leq 4\}$

(c) the x-intercepts: $(-6, 0), (0, 0)$

(d) $f(3) = 2$ (e) $f(-7) =$ does not exist

TO BE CONT'D

1.3 Continued

• Finding Information About a Graph.

EX: Given $f(x) = \dfrac{x^2+2}{x+4}$.

FIND: (a) Will the point $(1, \frac{3}{5})$ be on the graph of f?

$$\frac{3}{5} \stackrel{?}{=} \frac{1^2+2}{1+4}$$

$$\frac{3}{5} = \frac{3}{5} \Rightarrow \text{True} \Rightarrow \text{Yes}$$

(b) What is $f(0)$?

$$f(x) = \frac{x^2+2}{x+4}$$

$$f(0) = \frac{2}{4} = \underline{\frac{1}{2}}$$

(c) What point did we find in (b)?

$$\underline{(0, \tfrac{1}{2})} \Rightarrow \underline{\text{y-int}}$$

(d) If $f(x) = \frac{1}{2}$, what is x?

$$\cancel{2}(x+4)\frac{1}{\cancel{2}_1} = \frac{x^2+2}{\cancel{x+4}_1}(2)\cancel{(x+4)}^1$$

$$x + 4 = 2x^2 + 2$$

$$0 = 2x^2 - x$$

$$0 = x(2x-1)$$

$$\underline{x = 0} \quad \text{OR} \quad 2x - 1 = 0$$

$$2x = 1$$

$$\underline{x = \frac{1}{2}}$$

(e) Find the x-intercepts. Let $f(x) = 0$

$$0 = \frac{x^2+2}{x+4}$$

$$0 = x^2 + 2$$

$$-2 = x^2$$

$$\pm\sqrt{-2} = x = \pm\sqrt{2}\,i$$

$$\Rightarrow \underline{\text{No x-int}}$$