MWF 12, F19

2.5 Graphs of Other Functions

• Tangent

As we stated before, $\tan x = \frac{\sin x}{\cos x}$.

Note that the tangent is undefined when $\cos x = 0$. Therefore, the graph has vertical asymptotes for values of x where the $\cos x = 0$. These angles are at

$x = \frac{\pi}{2} + n\pi$, where n is any odd integer.

We also know that the fundamental period for the tangent is $T = \pi$.

EX: Graph $f(x) = \tan x$.

x	$\tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined

Observations:

From the function, $|A| = 1$

From the graph: $T = \pi$

$D_f = \{x \mid x \neq \frac{\pi}{2} + n\pi, n = \text{odd integer}\}$

$R_f = (-\infty, \infty)$, f is odd.

• Cotangent

EX: Graph $f(x) = \cot x$.

We know that $\cot x = \frac{\cos x}{\sin x} \Rightarrow$

vertical asymptotes at $x = n\pi$, n = integer

x	$\cot x$
0	undefined
$\pi/4$	1
$\pi/2$	0
$3\pi/4$	-1
π	undefined

Observations:

$|A| = 1$, $T = \pi$

$D_f = \{x \mid x \neq n\pi, n = \text{integer}\}$

$R_f = (-\infty, \infty)$, odd function

• Cosecant

EX: Graph $f(x) = \csc x$; $\csc x = \frac{1}{\sin x}$.

Vertical asymptotes at $\sin x = 0$

$D_f = \{x \mid x \neq n\pi, n = \text{integer}\}$

$R_f = (-\infty, -1] \cup [1, \infty)$, odd function

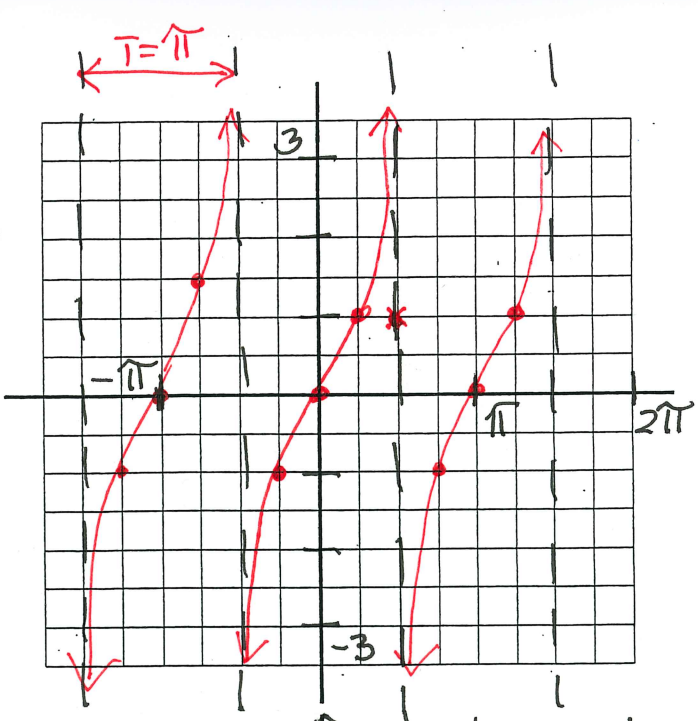
• Secant

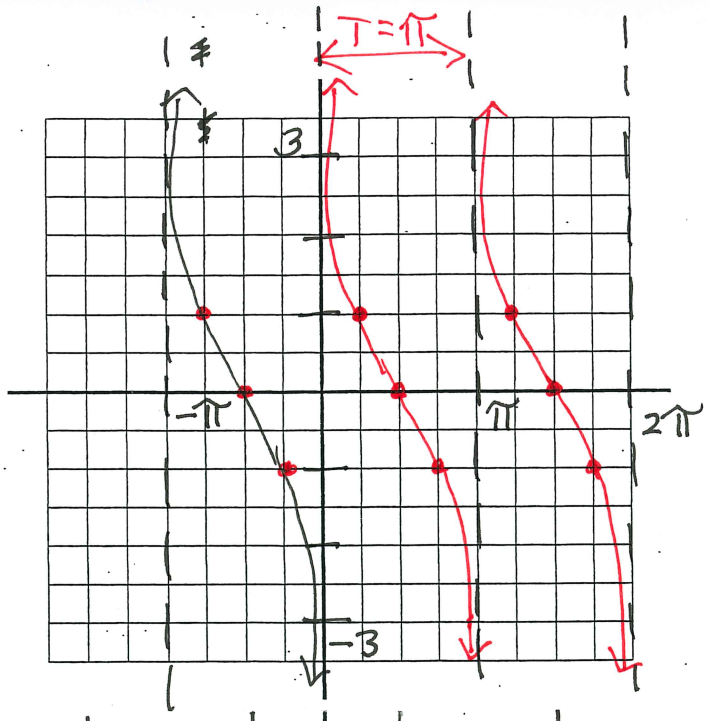
EX: Graph $f(x) = \sec x$; $\sec x = \frac{1}{\cos x}$

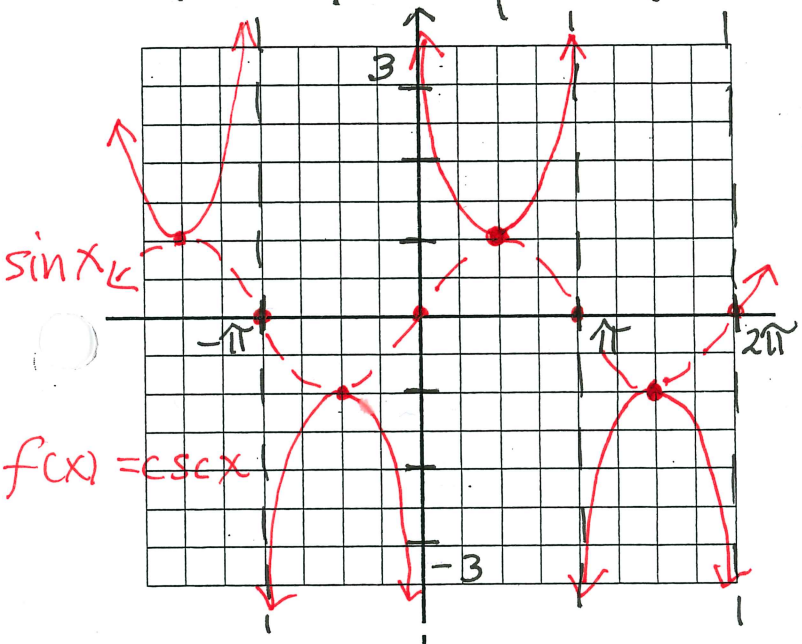
$\Rightarrow$ vertical asymptotes where $\cos x = 0$.

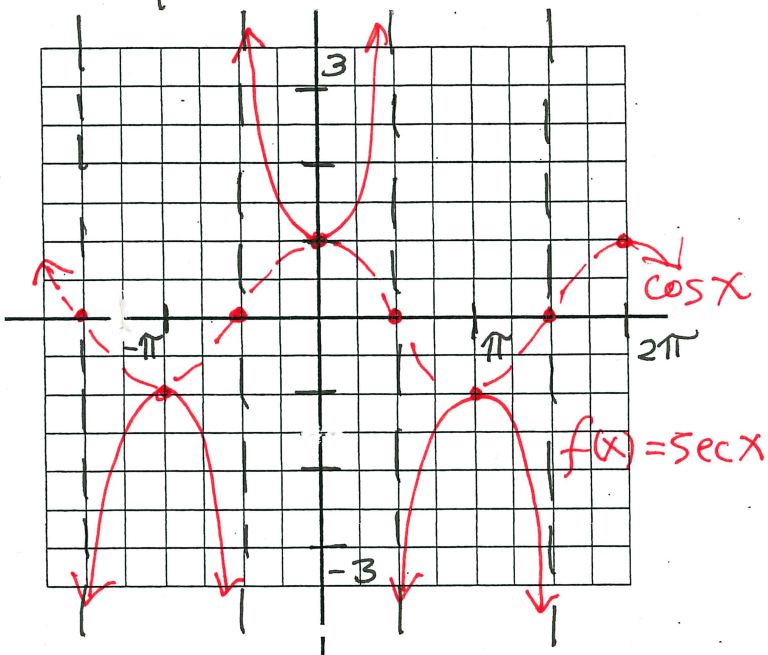
$D_f = \{x \mid x \neq \frac{\pi}{2} + n\pi, n = \text{odd integer}\}$

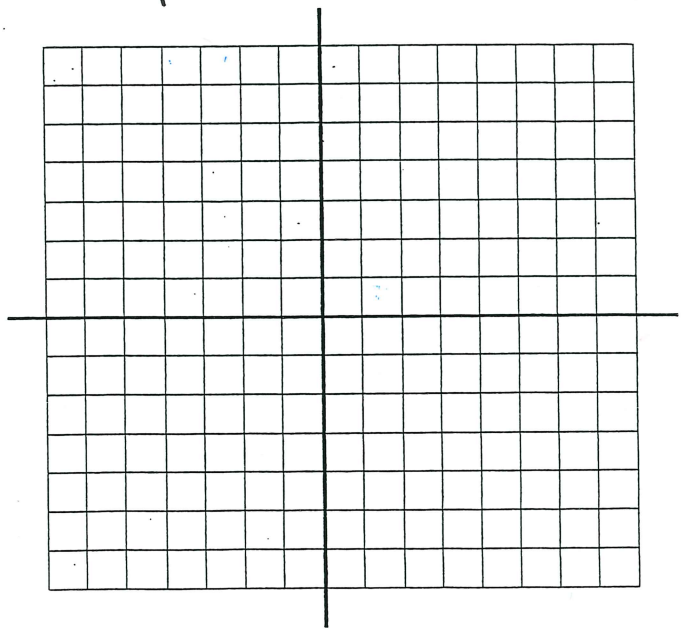
$R_f = (-\infty, -1] \cup [1, \infty)$, even function

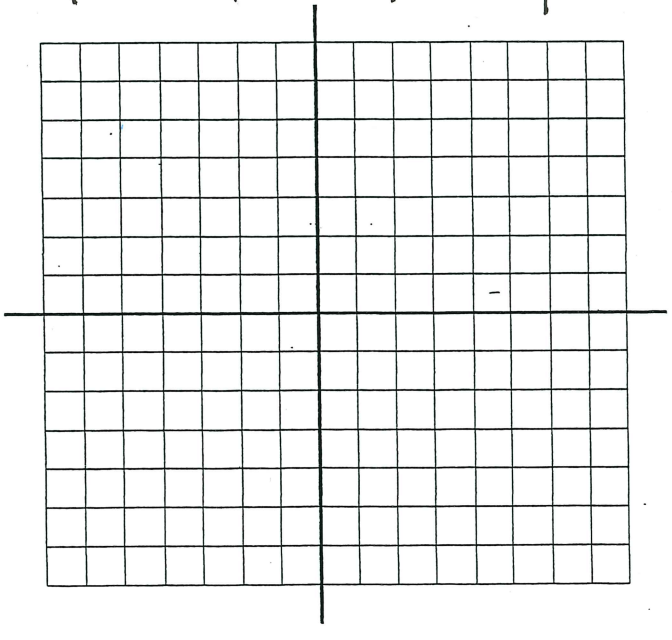

T = π
3
-π
π
2π
-3


T = π
3
-π
π
2π
-3


sin x
3
-π
π
2π
-3
f(x) = csc x


cos x
3
-π
π
2π
-3
f(x) = sec x



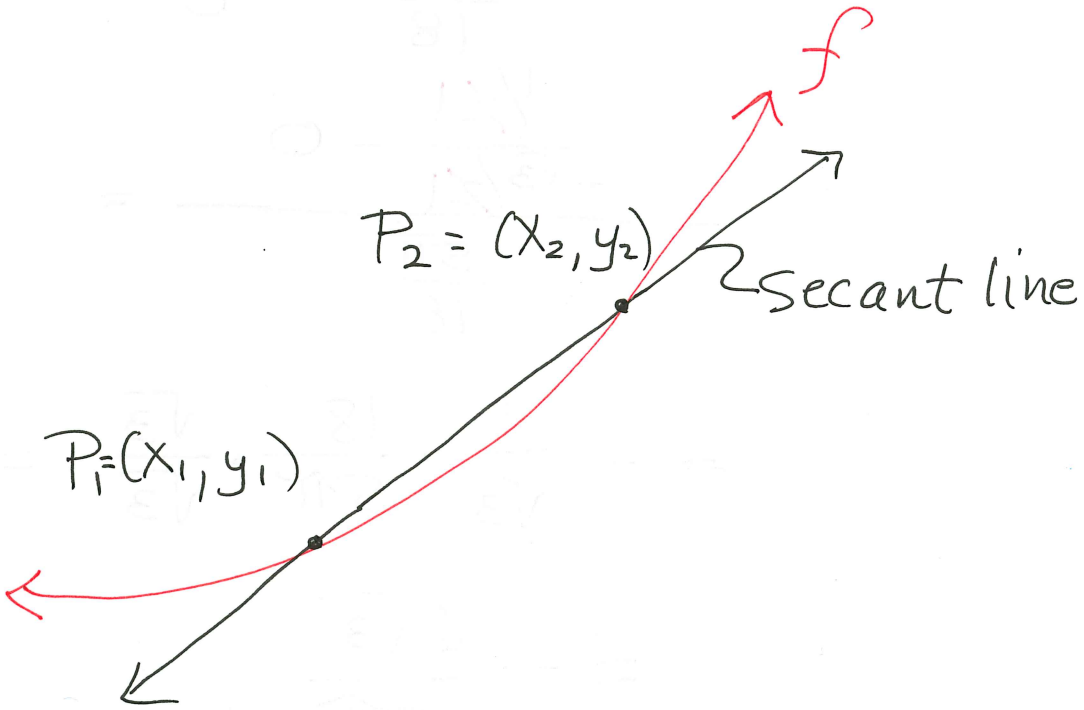


- ## Average Rate of Change

Recall from Algebra:

$$ROC_{avg.} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = m_{sec}$$

change in



EX: Find $ROC_{avg.}$ for $f(x) = 3x^2$ from $x_1 = 1$ to $x_2 = 5$.

$$ROC_{avg.} = \frac{f(5) - f(1)}{5 - 1} = \frac{75 - 3}{4} = \frac{72}{4}$$

$$= 18 = m_{sec}$$

EX: Find ROC_{avg} for $g(x) = \tan(3x)$, from $x_1 = 0$ to $x_2 = \frac{5\pi}{18}$.

$$ROC_{avg} = \frac{\tan\left(3 \cdot \frac{5\pi}{18}\right) - \tan 0}{\frac{5\pi}{18} - 0}$$

$$= \frac{\frac{1/2}{-\sqrt{3}/2} - 0}{\frac{5\pi}{18}} = \frac{-\frac{1}{\sqrt{3}}}{\frac{5\pi}{18}}$$

$$= -\frac{1}{\sqrt{3}} \cdot \frac{18}{5\pi} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{5\pi}$$

$$= \boxed{-\frac{6\sqrt{3}}{5\pi}}$$