1.7 One-to-One Functions; Inverse Functions

Def - a function on the xy-plane is said to be one-to-one if no two ordered pairs have the same y-coordinate.

- Horizontal Line Test - if any horizontal line intercepts the graph of a function at most at one point, then the function is 1-1.

  If a function is 1-1 it has an inverse function.

Def - If f and g are inverse functions of each other, then f and g must be 1-1 if and only if $f(g(x)) = x = g(f(x))$. This is called the composition property of functions,

Def - If f is 1-1, the inverse function of f is identified by f^{-1} (inverse of f).

EX: Use the <u>composition property</u> to show that $f(x) = \frac{x+1}{2}$ and $g(x) = 2x-1$ are inverse functions of each other.

$$(f \circ g)(x) = f(g(x)) = f(2x-1) = \frac{(2x-1)+1}{2} = \frac{2x}{2} = \underline{x}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x+1}{2}\right) = \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1 = \underline{x}$$

Therefore, f and g are inverses of each other.

NOTE: The <u>domain</u> of a function f is equal to the <u>range</u> of f^{-1}, and the <u>range</u> of f is equal to the <u>domain</u> of f^{-1}!

- <u>Finding Inverse Functions Algebraically</u>

EX: Given $f(x) = \frac{2x+1}{3}$. Find f^{-1}!

(1) Make sure that f is 1-1.

f is linear whose graph meets the horizontal line test.

(2) Write the function in the form $y = f(x)$.

$$y = \frac{2x+1}{3}$$

(3) Interchange x and y: $x = \frac{2y+1}{3}$

(4) Solve for y:

$$3x = 2y + 1$$
$$3x - 1 = 2y$$
$$\frac{3x-1}{2} = y$$

(5) Replace y with $f^{-1}(x)$:

$$\underbrace{f^{-1}(x) = \frac{3x-1}{2}}$$

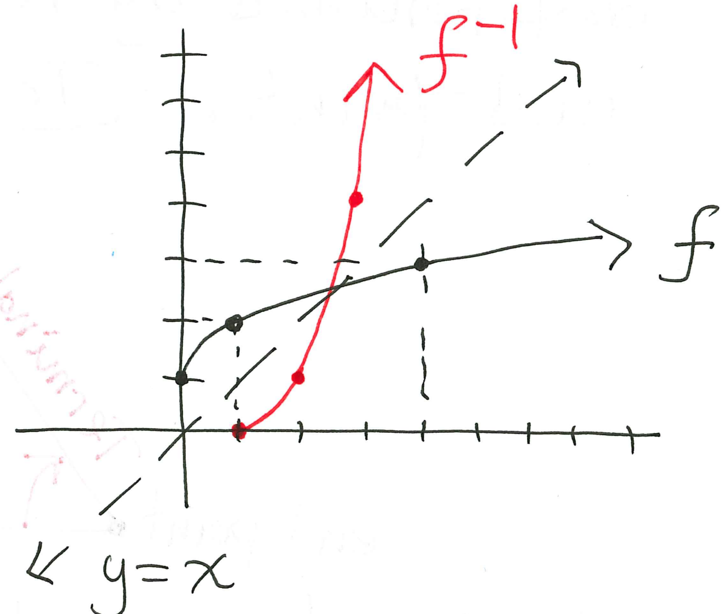
- <u>Finding the Inverse Graphically</u>

<u>EX</u>: Given the graph of f. Find graph of f^{-1}.

x	f(x)
0	1
1	2
4	3

$\Rightarrow$

x	$f^{-1}(x)$
1	0
2	1
3	4



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