MWF 12, F19

2.6 Phase Shift and Applications

• General Form of Sine and Cosine Functions

horizontal stretch or shrink

related to horizontal shifts

$$f(x) = A \sin(\omega x - \phi) + B$$

A: vertical stretch or shrink; neg. sign indicates a reflection on x-axis

B: vertical shifts

NOTE: $\sin(\omega x - \phi) = \sin\left(\omega\left(x - \frac{\phi}{\omega}\right)\right)$

This ratio of $\frac{\phi}{\omega}$ is called the phase shift and

it is to the right $\frac{\phi}{\omega} > 0$ or

it is to the left $\frac{\phi}{\omega} < 0$

EX: Find the amplitude, period, and phase shift for $g(x) = -3 \sin\left(-2x - \frac{\pi}{2}\right)$.

Using the fact that the sine function is odd

$$\Rightarrow g(x) = 3 \sin\left(2x + \frac{\pi}{2}\right) *$$

Then, $|A| = 3$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

$\frac{\phi}{\omega} = \frac{-\pi/2}{2} = -\frac{\pi}{2} \cdot \frac{1}{2} = -\frac{\pi}{4}$, left phase shift

EX: Write an equation of a sine function with the following properties:

$|A| = 5$, $T = \frac{\pi}{2}$, $\frac{\phi}{\omega} = 2$.

$T = \frac{2\pi}{\omega}$

$\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{\pi/2}$

$= 2\pi \cdot \frac{2}{\pi}$

$\omega = 4$

$\Rightarrow \phi = 2\omega = 2(4)$

$\phi = 8$

$\Rightarrow f(x) = 5\sin(4x - 8)$

- Applications

EX: Suppose the monthly average high temperatures for SA are used to find the sinusoidal model:

$$y = 16.95 \sin(0.48x - 1.79) + 78.16.$$

For this function, x represents the months of the year and y corresponds to each averege high monthly temperature.

FIND: The amplitude and the period to 2 decimals.

$|A| = 16.95$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.48} = 13.09$ months

EX: the monthly average high temperatures for Qualliyute, WA, are used to find the sinusoidal function that best fits the data:

$$Q(t) = 11.08 \sin(0.55t - 2.44) + 57.87,$$

where t represents the month of the year and Q the corresponding average high temperature.

FIND: (a) $|A|, \omega, T, \phi, \frac{\phi}{\omega}$.

$|A| = 11.08$, $\omega = 0.55$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.55} = 11.42$ months

$\phi = 2.44$, $\frac{\phi}{\omega} = \frac{2.44}{0.55} = 4.44$ months shifted right

(b) the average high temperature for July to one decimal.

$$Q(7) = 11.08 \sin(0.55(7) - 2.44) + 57.87$$

$$Q(7) = 68.8\,^\circ F$$