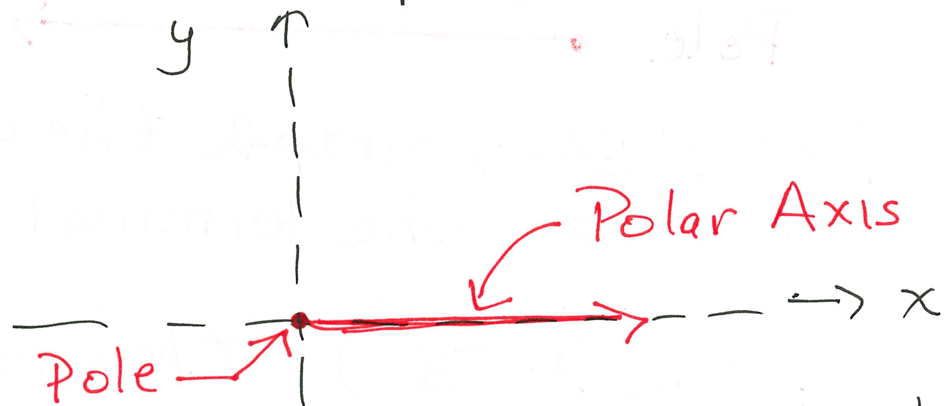
# 5.1 Polar Coordinates

In the polar coordinate system, we select a point, called the pole, and then a ray with its vertex at the pole. This ray is called the polar axis.

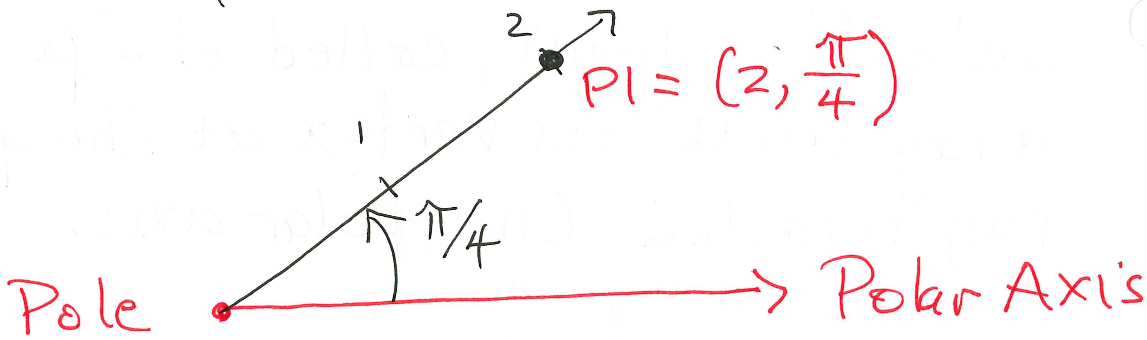



- **Plotting Points Using Polar Coordinates**

A point P in polar coordinate form is an ordered pair, $(r, \theta)$.

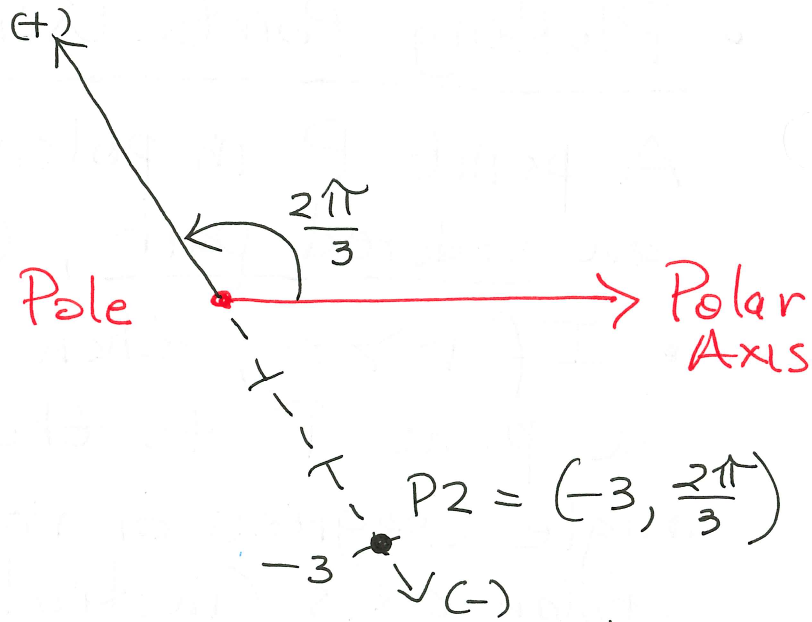
- If $r > 0$, then $r$ is the distance of a point P to the pole. $\theta$ is the angle (degrees or radians) formed by the polar axis (initial side) and a ray from the pole to the point P (terminal side).
- If $r < 0$, the point $(r, \theta)$ is on a ray from the pole extending in the opposite direction from the terminal side of $\theta$ a distance $|r|$ from the pole.

For example, if $P1 = (2, \frac{\pi}{4})$, then its graph is as follows:



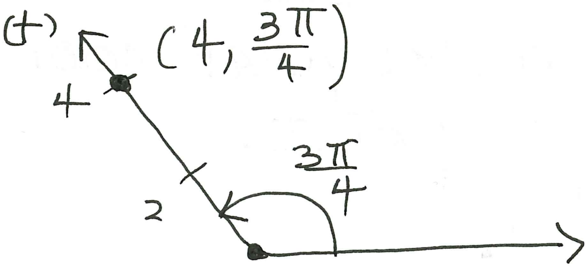

NOTE: We always graph the angle $\theta$ first, and then the terminal side r.

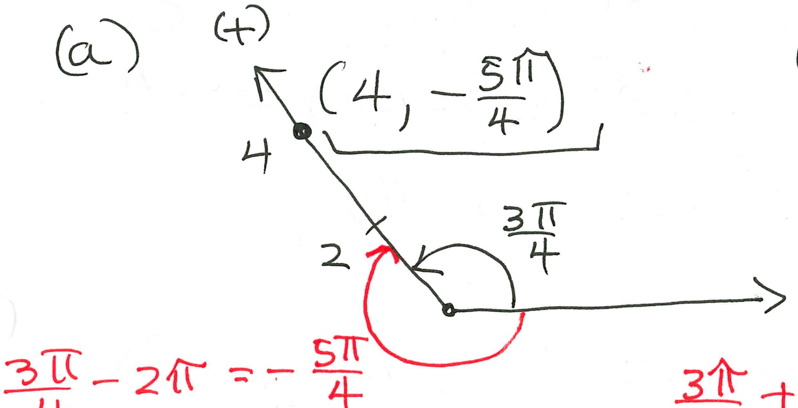
If $P2 = (-3, \frac{2\pi}{3})$, then its graph is the following:

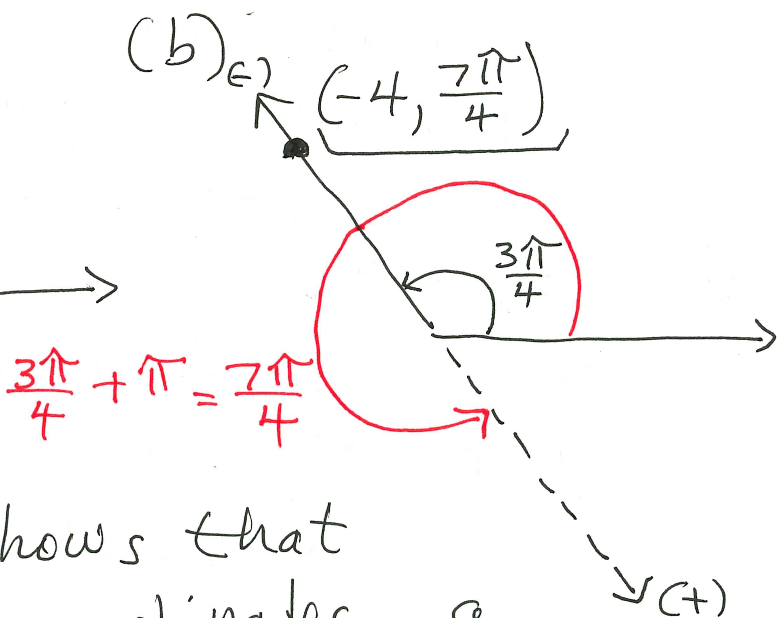



As was the case with the Unit Circle, we can find many ordered pairs for a particular point on the polar coordinate system by adding/subtracting multiples of $2\pi$.

EX: Plot the point $(4, \frac{3\pi}{4})$ and find other coordinates, $(r, \theta)$, for this same point for which (a) $r > 0$, $-2\pi \le \theta < 0$, and (b) $r < 0$, $0 \le \theta \le 2\pi$.




This example also shows that unlike rectangular coordinates, a point in polar form can have an <u>infinite number of ordered pairs</u>. Thus, any point $(r, \theta)$ can be written in <u>general form</u> as:

$$(r, \theta + 2\pi k) \text{ or } (-r, \theta + \pi + 2\pi k)$$

where $k$ is an integer. <u>Note</u> that the pole has coordinates $(0, \theta)$.

## Converting from Polar to Rectangular Form

Def- If P is a point will polar coordinates $(r, \theta)$, the rectangular coordinates of P are given by: $\boxed{x = r\cos\theta \text{ and } y = r\sin\theta.}$

EX: Find the rectangular coordinates for each given point.

(a) $\left(4, \frac{3\pi}{2}\right)$.

$$x = r\cos\theta = 4\cos\frac{3\pi}{2} = 4(0) = 0$$

$$y = r\sin\theta = 4\sin\frac{3\pi}{2} = 4(-1) = -4$$

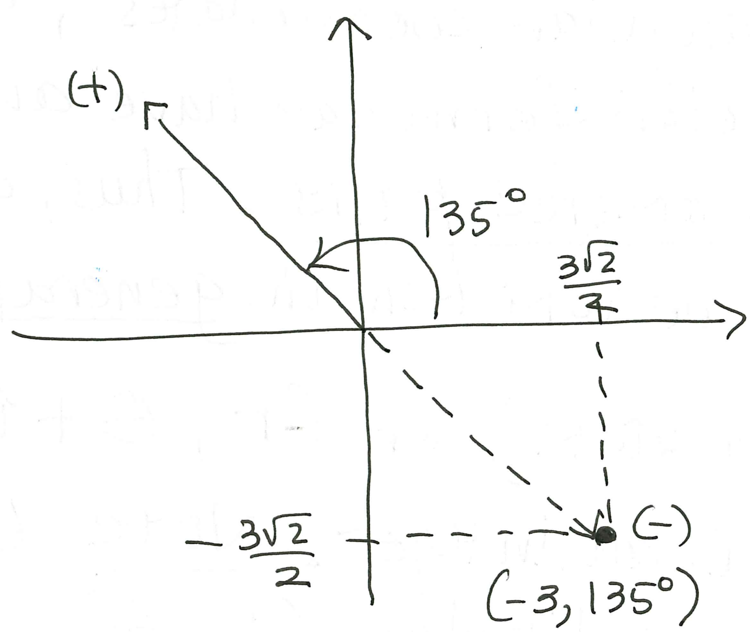
$$\left(4, \frac{3\pi}{2}\right) \Leftrightarrow \underbrace{(0, -4)}$$

(b) $(-3, 135^\circ)$.

$$x = r\cos\theta = -3\cos 135^\circ = -3\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}$$

$$y = r\sin\theta = -3\sin 135^\circ = -3\left(\frac{\sqrt{2}}{2}\right) = -\frac{3\sqrt{2}}{2}$$

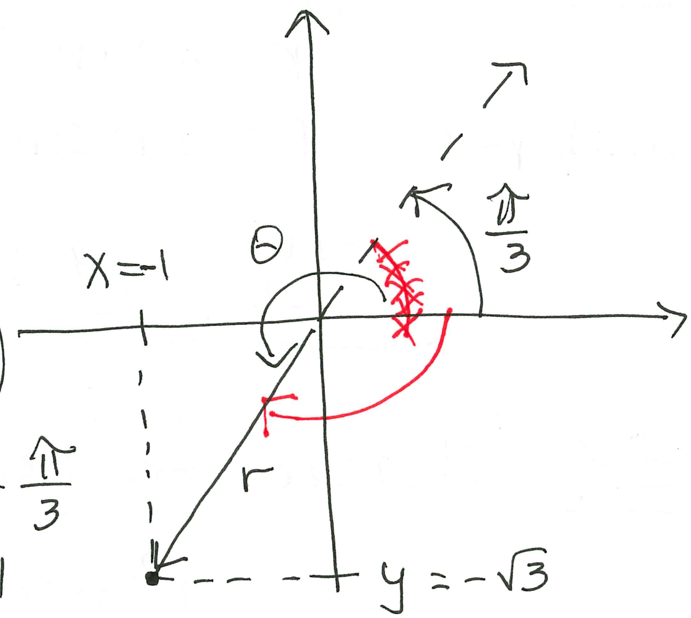
$$(-3, 135^\circ) \Leftrightarrow \underbrace{\left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)}$$



## • Converting From Rectangular to Polar Form

This process beomes <u>more</u> complicated than the previous one. We <u>always</u> start by <u>sketching</u> the rectangular coordinates given and use them to guide us to the polar form.

<u>EX:</u> Find the polar coordinates for the point $(-1, -\sqrt{3})$. <u>TO BE CONT'D</u>




$r = \sqrt{x^2+y^2} = \sqrt{1+3}$

$\underline{r = 2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$, $\theta$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

But, $\theta$ must be in Q3!!

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \Rightarrow \underline{\left(2, \frac{4\pi}{3}\right)}$

Other answers $\underline{\left(-2, \frac{\pi}{3}\right)}$ OR $\theta = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$

$\Rightarrow \underline{\left(2, -\frac{2\pi}{3}\right)}$

To summarize these results:

(1) <u>Always</u> graph $(x, y)$ on a rectangular grid.

(2) If $y$ or $x$ equals zero, use the graph to find $r$ and $\theta$.

(3) If $x \neq 0$ or $y \neq 0$, then $r = \sqrt{x^2+y^2}$.

(4) Determine the quadrant for $\theta$ from the graph and then use one of the following;

- if $\theta$ is in Q1 or Q4, then $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.
- if $\theta$ is in Q2 or Q3, then $\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$,

• Transform Equations

EX: Change to rectangular form.

$$r = \sin\theta + 1$$

Mult. r: $r^2 = r\sin\theta + r$

$$x^2 + y^2 = y + \sqrt{x^2 + y^2}$$

EX: Change to polar form.

$$y^2 = 2x$$

$$(r\sin\theta)^2 = 2r\cos\theta$$

$$r^2\sin^2\theta = 2r\cos\theta$$

$$r\sin^2\theta = 2\cos\theta$$

$$r\sin^2\theta - 2\cos\theta = 0$$