

5.2 Polar Equations and Their Graphs

To graph a polar equation we use a grid of concentric circles with the center at the pole and rays with their vertices also at the pole (see handout).

In this section we will graph and identify simple polar equations by converting them to rectangular form and graphing them using Algebraic methods. The general method of calculating ordered pairs, (r, θ) , to draw polar graphs will be done later in Calculus.

EX: Identify and graph the polar equation $r = 10$.

First change to rectangular form using:

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2, \quad \& \quad \tan \theta = \frac{y}{x}.$$

$$\text{Then, } r = 10$$

$$r^2 = 100$$

$$\underline{x^2 + y^2 = 100}$$

This equation graphs a circle centered at $(0, 0)$ with radius $r = 10$.

EX: Identify and graph the polar equation $\theta = \frac{7\pi}{4}$.

$$\tan \theta = \frac{y}{x} \Rightarrow \tan \frac{7\pi}{4} = -1$$

$$\Rightarrow \frac{y}{x} = -1$$

$$y = -x$$

This equation graphs a line with a slope $m = -1$ and y-int at $(0, 0)$.

EX: Identify and graph $r \csc \theta = 4$.

$$\frac{r}{\sin \theta} = 4$$

$$r = 4 \sin \theta$$

Mult. by r : $r^2 = 4r \sin \theta$

$$x^2 + y^2 = 4y$$

$$x^2 + y^2 - 4y + 4 = 0 + 4$$

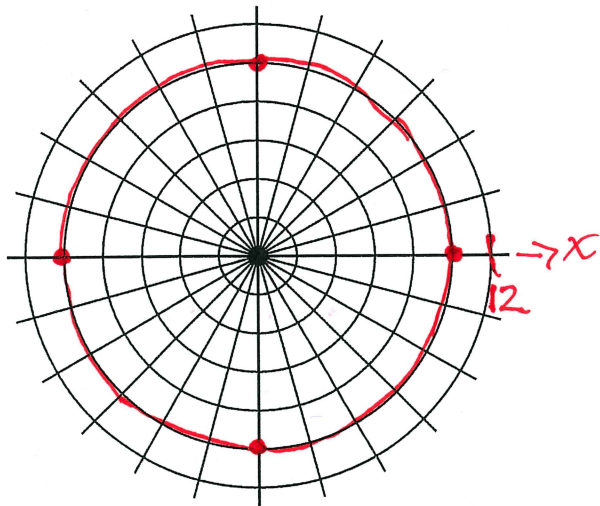
$$x^2 + (y - 2)^2 = 4$$

This equation graphs a circle centered at $(0, 2)$ with radius $r = 2$

Recall that $(x - h)^2 + (y - k)^2 = r^2$ is the standard form for the equation of a circle with radius r and centered at a point (h, k) .

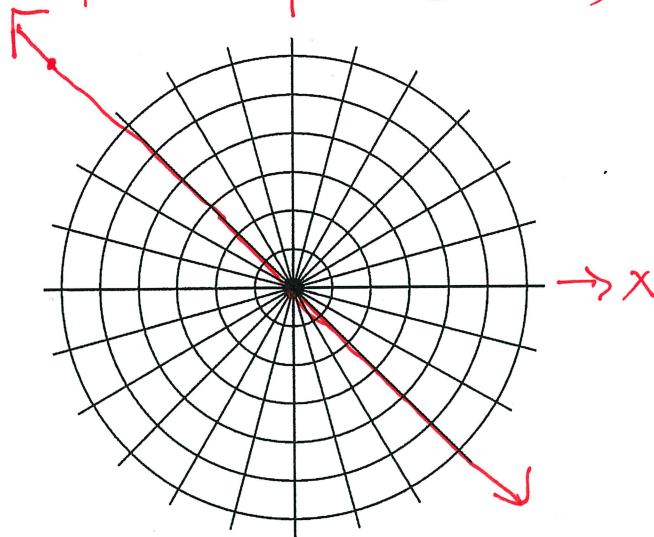
$$r = 10$$

$$y \uparrow \quad (x^2 + y^2 = 100)$$



$$\Theta = \frac{2\pi}{4}$$

$$y \uparrow \quad (y = -x)$$



$$r = \csc \Theta = 4$$

$$y \uparrow \quad [x^2 + (y-2)^2 = 4]$$

