MWF

3.7 Product-to Sum & Sum-to-Product Formulas

- **Product-to-Sum Identities**

$$\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\sin\alpha \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\cos\alpha \sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

- **Sum-to-Product Formulas**

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

These formulas are also found in the List of Trig. Formulas in Handout.

EX: Write the $\sin(2x)\cos(3x)$ as a sum/diff.

Let $\alpha = 2x$, $\beta = 3x$

Then, $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

$= \frac{1}{2}\left[\sin(5x) + \underset{(\text{odd})}{\sin(-x)}\right] = \frac{1}{2}\left[\sin(5x) - \sin x\right]$

$= \frac{1}{2}\sin(5x) - \frac{1}{2}\sin x$

EX: Express the given sum as a product of sines and/or cosines.

$\cos(2\theta) + \cos(8\theta)$; let $\alpha = 2\theta$, $\beta = 8\theta$

Then, $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$= 2\cos(5\theta)\cos(-3\theta)$

(even)

$= 2\cos(5\theta)\cos(3\theta)$

EX: Find the exact value for $\sin 165^\circ \cos 75^\circ$.

Let $\alpha = 165^\circ$, $\beta = 75^\circ$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$= \frac{1}{2}[\sin 240^\circ + \sin 90^\circ] = \frac{1}{2}\left[-\frac{\sqrt{3}}{2} + 1\right]$

EX: Find exact value for $\cos 195^\circ + \cos 105^\circ$.

Let $\alpha = 195^\circ$, $\beta = 105^\circ$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$= 2\cos 150^\circ \cos 45^\circ$

$= \cancel{2}^{1}\left(-\frac{\sqrt{3}}{\cancel{2}_{1}}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\cos 195^\circ + \cos 105^\circ = -\frac{\sqrt{6}}{2}$

EX: Establish $\sin(5\theta) - \sin(3\theta) = 2\sin\theta\cos(4\theta)$.

Start with left side: let $\alpha = 5\theta$, $\beta = 3\theta$

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$= 2\sin\theta\cos(4\theta) \quad \text{(RS)}$$

END CH. 3 !