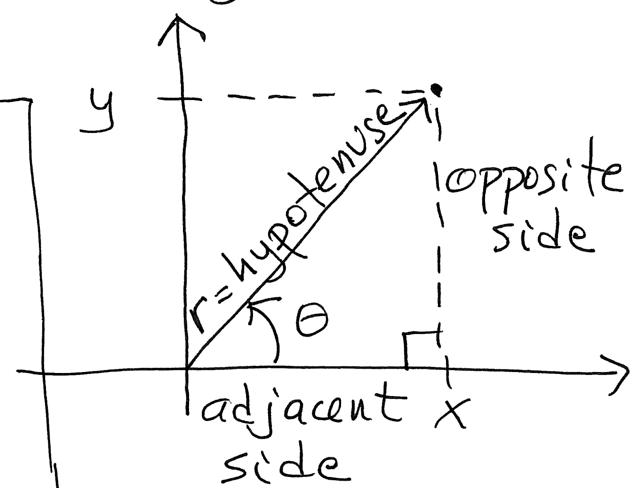
4.1 Applications of Right Triangles

We can also define all the trig. functions for the special case of an angle in QI using a right triangle.

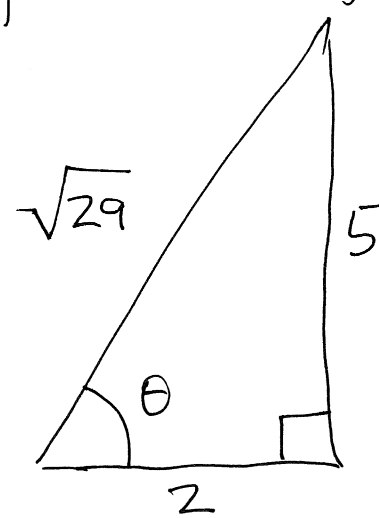


$$\sin\theta = \frac{y}{r} = \frac{\text{opp.}}{\text{hyp.}}, \quad \csc\theta = \frac{r}{y} = \frac{\text{hyp.}}{\text{opp.}}$$

$$\cos\theta = \frac{x}{r} = \frac{\text{adj.}}{\text{hyp.}}, \quad \sec\theta = \frac{r}{x} = \frac{\text{hyp.}}{\text{adj.}}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{opp.}}{\text{adj.}}, \quad \cot\theta = \frac{x}{y} = \frac{\text{adj}}{\text{opp.}}$$

EX: Evaluate the 6 trig. functions for the given triangle.

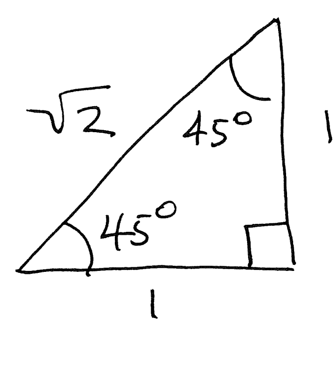


$$\sin\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}, \quad \csc\theta = \frac{\sqrt{29}}{5}$$

$$\cos\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}, \quad \sec\theta = \frac{\sqrt{29}}{2}$$

$$\tan\theta = \frac{5}{2}, \quad \cot\theta = \frac{2}{5}$$

- Special Triangles
  - 45° Triangle



$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos 45^\circ$$

$$\tan 45^\circ = 1$$

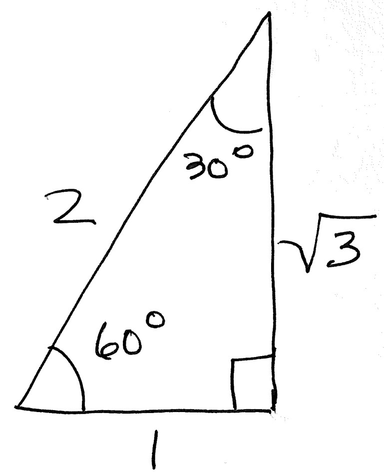
— 30°–60°

$\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$



Because 30° and 60° are complementary angles, then $\sin 30^\circ = \cos 60^\circ$ and $\sin 60^\circ = \cos 30^\circ$.

TO BE CONT'D

4.1 Continued

These two functions are called cofunctions of complementary angles.

- Complementary Angles Theorem

Cofunctions of complementary angles are equal. that means that if θ is an acute angle of a right triangle, then the following are true:

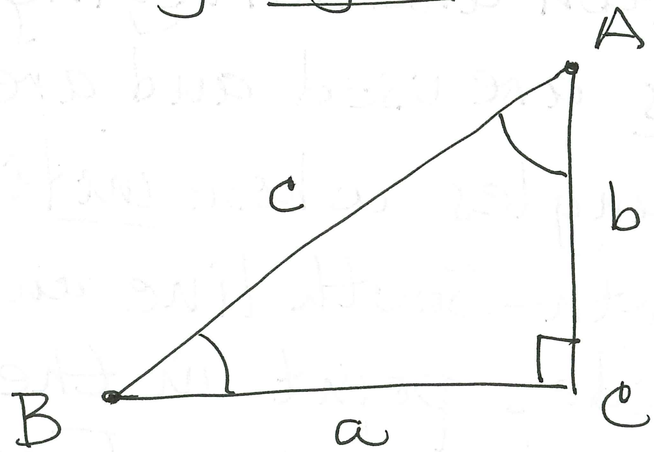
$\sin\theta = \cos(90^\circ - \theta)$, $\cos\theta = \sin(90^\circ - \theta)$

$\tan\theta = \cot(90^\circ - \theta)$, $\cot\theta = \tan(90^\circ - \theta)$

$\sec\theta = \csc(90^\circ - \theta)$, $\csc\theta = \sec(90^\circ - \theta)$

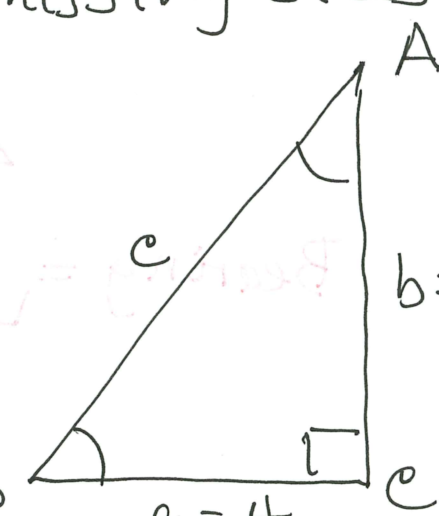
• Solving Right Triangles

To solve any right triangle, such as



means to find the measure of the missing angles, and the length of any missing sides. To do that we use (1) trig. functions, (2) $c^2 = a^2 + b^2$, and (3) $A + B = 90°$.

EX: Solve the given triangle. Round angle measures in degrees to one decimal and the lengths of any missing sides to two decimals.



$c^2 = a^2 + b^2$

$= 16 + 9 = 25$

$c = 5$

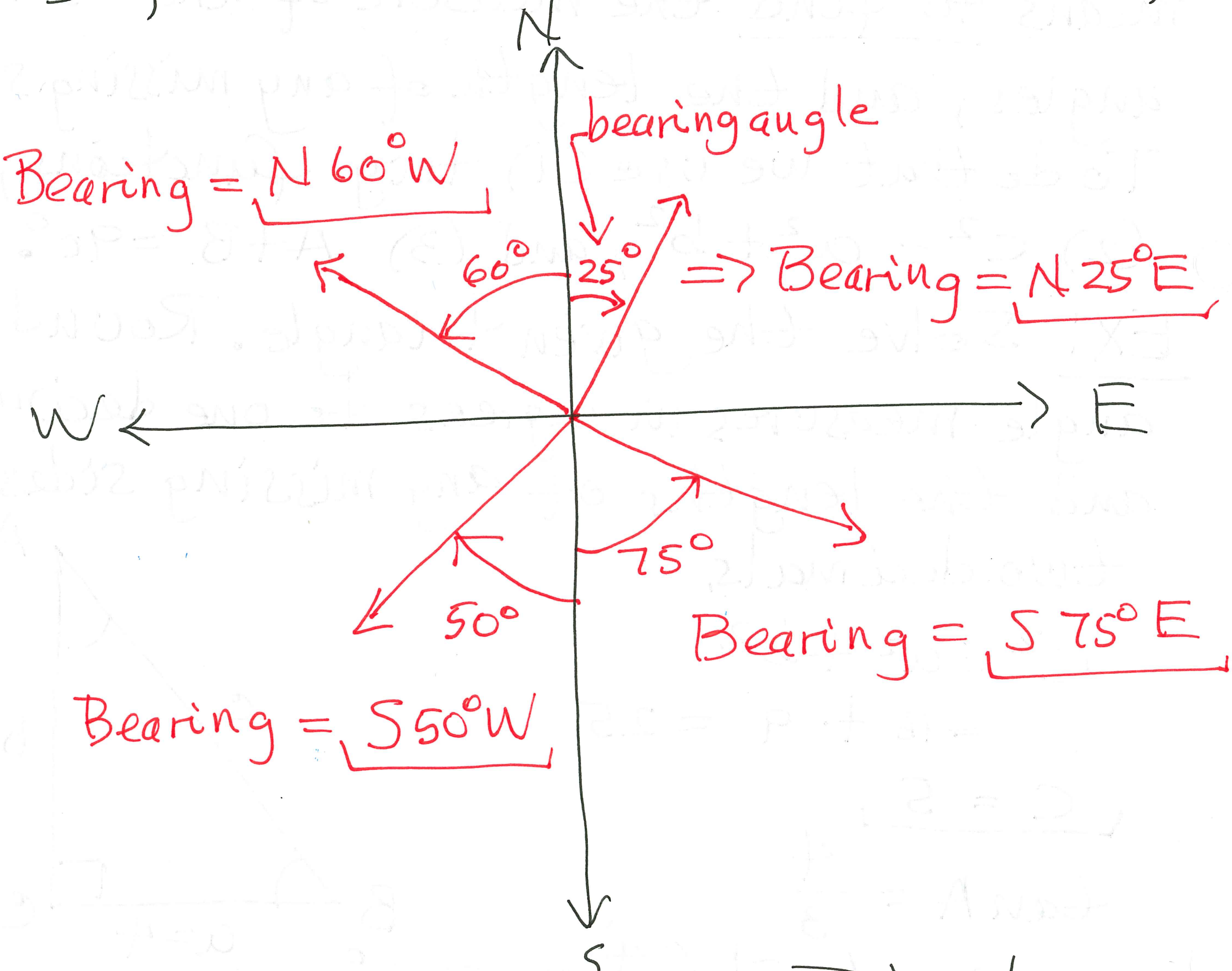
$\tan A = \frac{4}{3}$

$\Rightarrow A = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°$

$B = 90° - A = 90° - 53.13° = 36.9°$

- **Terminology for Navigation & Surveying**

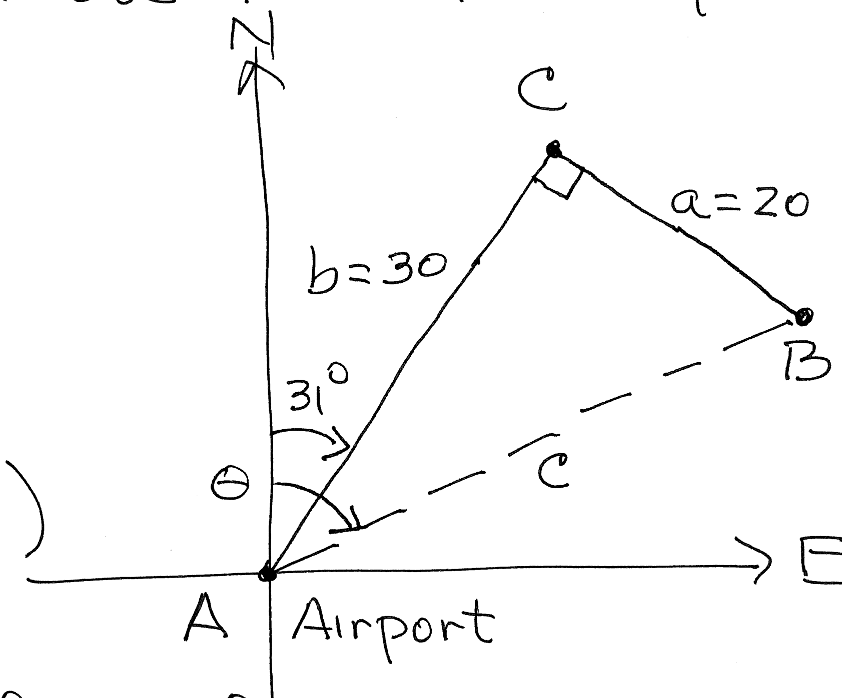
In navigation and surveying, directions or bearings are used and are defined by acute angles whose initial sides lie on the North–South line and whose terminal sides point in the NE, NW, SE, or SW directions. For example,



- **Applications of Right Triangles**

EX: An airplane leaves an airport from a runway whose bearing is N31°E. After flying 30 miles, the pilot turns the plane 90° to the right. The plane then flies an additional 20 miles in the new direction

(a) What bearing, to one decimal, should the control tower use to locate the plane?



$\theta = A + 31^\circ$

$\tan A = \frac{20}{30}$

$\Rightarrow A = \tan^{-1}\left(\frac{20}{30}\right)$

$= \arctan\left(\frac{20}{30}\right)$

$A = 33.69^\circ$

$\theta = 33.69^\circ + 31^\circ = 64.7^\circ$

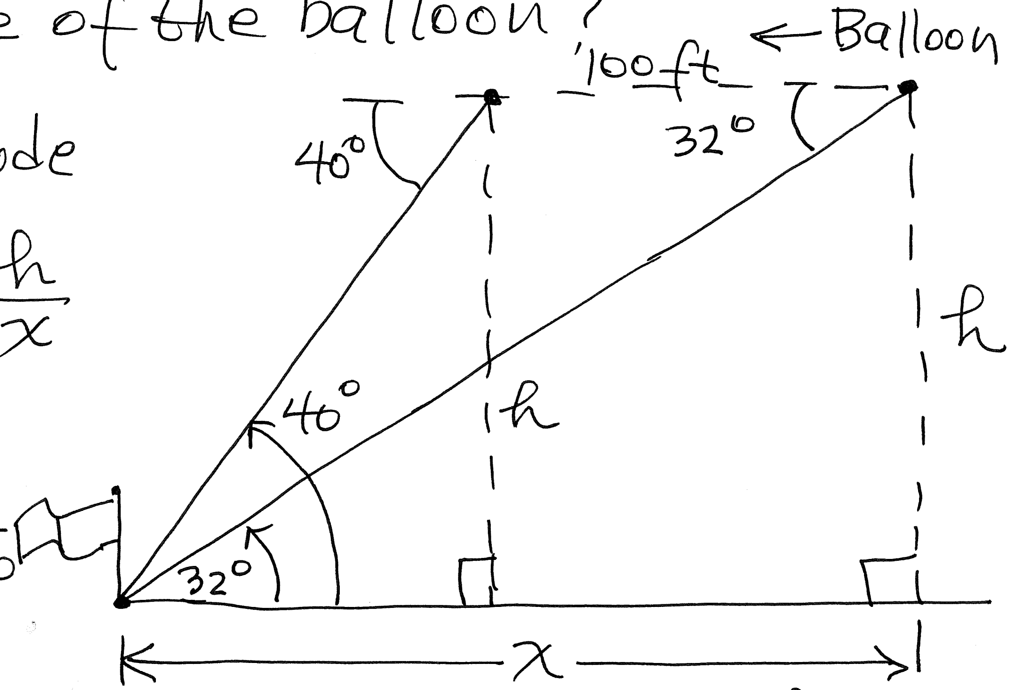
$\Rightarrow$ Bearing $= \underbrace{N64.7^\circ E}$

(b) What is the distance to the plane from the control tower at the airport to the plane, to one decimal?

$c^2 = a^2 + b^2 = 400 + 900 = 1300$

$c = \sqrt{1300} = \underbrace{36.1 \text{ miles}}$

<u>EX!</u> A hot air balloon is travelling at a constant altitude. At one point the angle of <u>depression</u> between the balloon and the base of a flag pole on the ground is $32°$. After travelling 100 ft, the angle of depression becomes $40°$. What is the altitude of the balloon?



Let h = altitude

(1) $\tan 32° = \dfrac{h}{x}$

(2) $\tan 40° = \dfrac{h}{x-100}$

(1) $x = \dfrac{h}{\tan 32°}$

(2) $x - 100 = \dfrac{h}{\tan 40°}$

$x = \dfrac{h}{\tan 40°} + 100$

$\Rightarrow \dfrac{h}{\tan 32°} = \dfrac{h}{\tan 40°} + 100$

$h \tan 40° = h \tan 32° + 100 \tan 32° \tan 40°$

$h \tan 40° - h \tan 32° = 100 \tan 32° \tan 40°$

$h(\tan 40° - \tan 32°) = 100 \tan 32° \tan 40°$

$h = \dfrac{100 \tan 32° \tan 40°}{\tan 40° - \tan 32°} = \underline{245 \text{ ft}}$