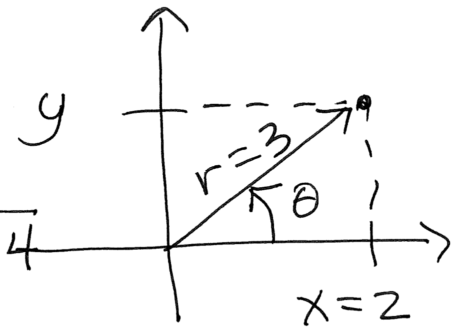
3.2 More Inverse Trig. Functions

- Different Compositions

EX: Find the exact value for $\tan\left(\cos^{-1}\frac{2}{3}\right)$.

Let $\theta = \cos^{-1}\frac{2}{3} \Rightarrow \cos\theta = \frac{2}{3}$, $0 \le \theta \le \pi$ (QII, QI)

Because $\cos\theta > 0$ (QI, QIV), θ must in QI, thus,



$\cos\theta = \frac{2}{3} = \frac{x}{r} \Rightarrow x = 2, r = 3$

$\tan\theta = \frac{y}{x} \Rightarrow y = \sqrt{r^2 - x^2} = \sqrt{9-4}$

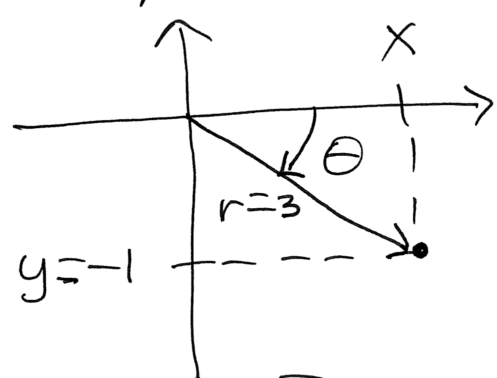
$y = \sqrt{5}$

$\tan\theta = \frac{\sqrt{5}}{2} \Rightarrow \underbrace{\tan\left(\cos^{-1}\frac{2}{3}\right) = \frac{\sqrt{5}}{2}}$

EX: Find the exact value for $\cos\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]$.

Let $\theta = \sin^{-1}\left(-\frac{1}{3}\right) \Rightarrow \sin\theta = -\frac{1}{3}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (QI, QIV)

Since $\sin\theta < 0$ (QIII, QIV), θ must be in QIV,



$\sin\theta = \frac{y}{r} = \frac{-1}{3} \Rightarrow y = -1, r = 3$

$\cos\theta = \frac{x}{r}$, $x = \sqrt{r^2 - y^2} = \sqrt{9-1}$ $y = -1$

$= \sqrt{8} = \underline{2\sqrt{2}}$

$\cos\theta = \frac{2\sqrt{2}}{3} \Rightarrow \underbrace{\cos\left[\sin^{-1}\left(-\frac{1}{3}\right)\right] = \frac{2\sqrt{2}}{3}}$

- <u>Definitions for Inverse Secant, Cosecant, and Cotangent Functions</u>

<u>Def</u>: (1) $y = \sec^{-1} x$ iff $x = \sec y$, where

$d_{\sec^{-1} x} = [-1, 1]$, $r_{\sec^{-1} x} = 0 \le y \le \pi,\ y \ne \frac{\pi}{2}$

(2) $y = \csc^{-1} x$ iff $x = \csc y$, where

$d_{\csc^{-1} x} = [-1, 1]$, $r_{\csc^{-1} x} = -\frac{\pi}{2} \le y < \frac{\pi}{2},\ y \ne 0$

(3) $y = \cot^{-1} x$ iff $x = \cot y$, where

$d_{\cot^{-1} x} = (-\infty, \infty)$, $r_{\cot^{-1} x} = 0 < y < \pi$

<u>EX</u>: Find the <u>exact</u> value for $\csc^{-1} 2$.

Let $\theta = \csc^{-1} 2 \Rightarrow \csc\theta = 2,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ \theta \ne 0$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \Rightarrow \underbrace{\csc^{-1} 2 = \frac{\pi}{6}}$

<u>EX</u>: Find the <u>exact</u> value for $\sec^{-1}(-2)$.

Let $\theta = \sec^{-1}(-2) \Rightarrow \sec\theta = -2,\ 0 \le \theta \le \pi,\ \theta \ne \frac{\pi}{2}$

$\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

$\Rightarrow \underbrace{\sec^{-1}(-2) = \frac{2\pi}{3}}$

• Using Calculator To Evaluate $\sec^{-1}x$, $\csc^{-1}x$, and $\cot^{-1}x$.

EX: (a) Find $\sec^{-1}3$.

Let $\theta = \sec^{-1}3 \Rightarrow \sec\theta = 3, 0 \le \theta \le \pi, \theta \ne \frac{\pi}{2}$

$\Rightarrow \cos\theta = \frac{1}{3} \Rightarrow \cos^{-1}\frac{1}{3} = \theta = \underline{1.23}$

Thus, $\underline{\sec^{-1}3 = 1.23}$

NOTE that $\sec^{-1}3 \ne \frac{1}{\cos^{-1}3}$

(b) Find $\cot^{-1}(-3)$.

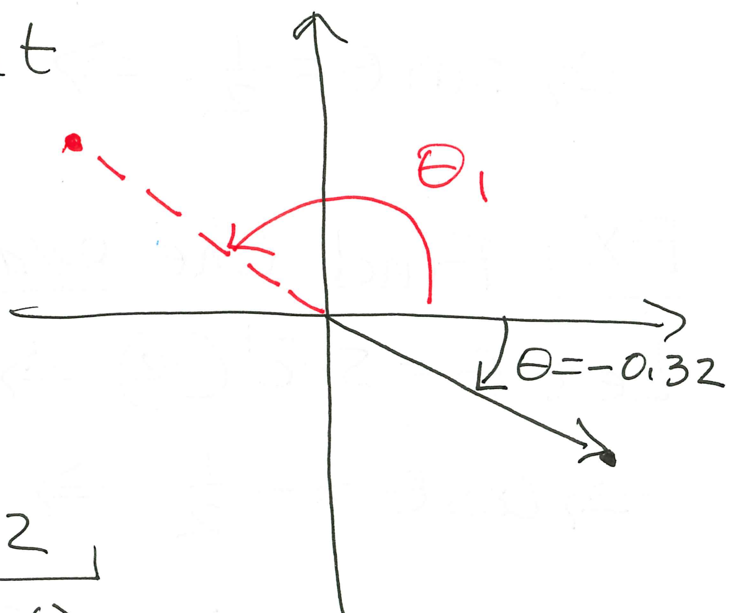
Let $\theta = \cot^{-1}-3 \Rightarrow \cot\theta = -3$, $0 < \theta < \pi$ (QI, QII)

$\tan\theta = -\frac{1}{3} \Rightarrow \tan^{-1}\left(-\frac{1}{3}\right) = -0.32$ in QIV

To find an equivalent angle in Q II

$\theta_1 = \pi - 0.32$

$\theta_1 = 2.82$



$\Rightarrow \underline{\cot^{-1}(-3) = 2.82}$

TO BE CONT'D

3.2 Continued

MWF 12, F19

- Writing Trig. Expressions as Algebraic Expressions

EX: Write $\tan(\cos^{-1} u)$ as an Algebraic expression in terms of u.

M.I. Using Fundamental Identies: ask MML to "view an example" when you start homework.

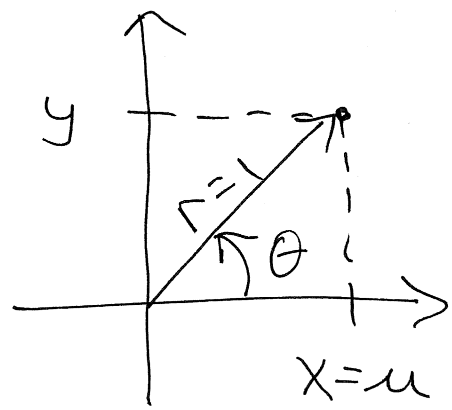
M.II. Using a circle of radius r:

Let $\theta = \cos^{-1} u \Rightarrow \cos\theta = u$

$\cos\theta = \frac{x}{r} = \frac{u}{1} \Rightarrow x = u, \; r = 1$

We want to find:

$\tan\theta = \frac{y}{x}$



$y = \sqrt{r^2 - x^2} = \sqrt{1-u^2}$

$\tan\theta = \frac{\sqrt{1-u^2}}{u}$

$$\Rightarrow \underbrace{\tan(\cos^{-1} u) = \frac{\sqrt{1-u^2}}{u}}$$

<u>EX</u>: Write $\tan(\sec^{-1} u)$ as an Algebraic expression in terms of u.

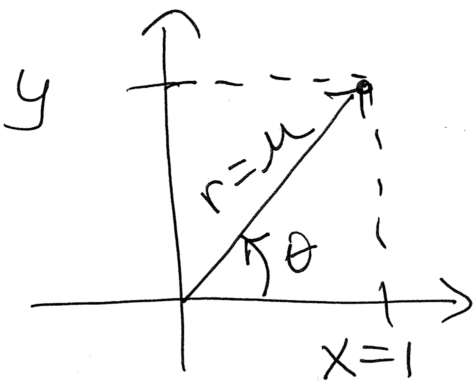
<u>I.</u> See MML - "view an example"

<u>II</u> <u>Using a circle of radius r</u>:

Let $\theta = \sec^{-1} u \Rightarrow \sec\theta = u = \frac{r}{x} = \frac{u}{1}$

$\Rightarrow u = r, \; x = 1$

$\tan\theta = \frac{y}{x}$



$y = \sqrt{r^2 - \cancel{x}^2} = \sqrt{u^2 - 1}$

$\underbrace{\tan\theta = \sqrt{u^2-1}}$

<u>EX</u>: Given $g(x) = \cos x$, $h(x) = \tan x$.

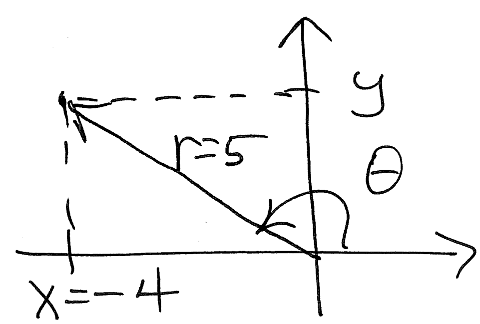
<u>FIND</u>: $h(g^{-1}(-\frac{4}{5}))$. $\Rightarrow \tan(\cos^{-1}(-\frac{4}{5}))$

Let $\theta = \cos^{-1}(-\frac{4}{5}) \Rightarrow \cos\theta = -\frac{4}{5}$, $0 \le \theta \le \pi$

Then, θ must be in Q2 $\Rightarrow \cos\theta = \frac{-4}{5} = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$, $y = \sqrt{25-16} = \sqrt{9}$

$\underline{y = 3}$



$\tan\theta = \frac{3}{-4}$

$= -\frac{3}{4}$

Thus, $\underbrace{h(g^{-1}(-\frac{4}{5})) = -\frac{3}{4}}$

EX: Given $f(x) = \sin x$, $g(x) = \cos x$.

FIND: $g^{-1}(f(-\frac{5\pi}{6}))$.

$$\Rightarrow \cos^{-1}\left(\sin\left(-\frac{5\pi}{6}\right)\right)$$

Using U.C., $\sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$

We also know, $y = \cos^{-1} x \iff x = \cos y$

where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$

$$\Rightarrow \cos^{-1}\left(-\frac{1}{2}\right) = \theta \Rightarrow \cos\theta = -\frac{1}{2}$$

Using U.C., we want an angle θ on $[0, \pi]$ whose cosine is $-\frac{1}{2}$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

Therefore, $\underline{g^{-1}\left(f\left(-\frac{5\pi}{6}\right)\right) = \frac{2\pi}{3}}$