

Chapter 3 - Analytic Trigonometry

MWF 2, F19

3.1 The Inverse of the Sine, Cosine, and Tangent Functions

Inverse of a Sine Function

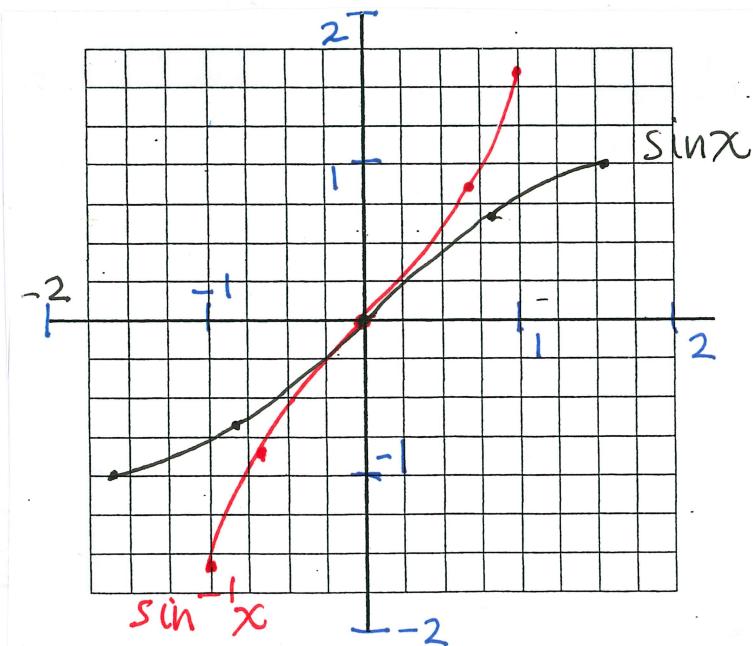
Is the function $f(x) = \sin x$ 1-1? NO!

However, if the domain is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ f will be 1-1 and has an inverse function, called the inverse sine function. It is written as:

$$f^{-1}(x) = \sin^{-1}x \text{ or arcsine } x.$$

The graph of this inverse function with its restricted domain can be sketched as:

x	$\sin x$	x	$\sin^{-1}x$
$-1.6 \approx -\frac{\pi}{2}$	-1	-1	-1.6
$-0.8 \approx -\frac{\pi}{4}$	-0.7	-0.7	-0.8
0	0	0	0
$0.8 \approx \frac{\pi}{4}$	0.7	0.7	0.8
$1.6 \approx \frac{\pi}{2}$	1	1	1.6



NOTE that $\sin^{-1}x$ is in fact an angle.

If $\sin^{-1}x = \theta$, then we know that this angle has a $\sin \theta = x$.

EX: $\sin \frac{\pi}{2} = 1 \Leftrightarrow \frac{\pi}{2} = \sin^{-1} 1 = \theta$

Def: $y = \sin^{-1}x$ iff $\sin y = x$,

$$d\sin^{-1}x = [-1, 1], r\downarrow \sin^{-1}x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

EX: Find the exact value for $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$. From U.C., $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$
We know that $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{3} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

EX: Find the exact value for $\sin^{-1}\left(-\frac{1}{2}\right)$.

Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right) \Rightarrow \sin \theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$
neither is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, but $\theta = -\frac{\pi}{6}$ is in the interval.

EX: Find the exact value for $\sin^{-1}\left(\sin \frac{5\pi}{6}\right)$.

Let $\theta = \sin^{-1}\left(\sin \frac{5\pi}{6}\right)$. We know that

$$f'(f(x)) = x \Rightarrow \theta = \frac{5\pi}{6}$$

But, $\frac{5\pi}{6}$ is NOT in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$$\sin \frac{5\pi}{6} = \sin \frac{\pi}{6} \Rightarrow \sin^{-1}\left(\sin \frac{5\pi}{6}\right) = \frac{\pi}{6}$$

Inverse Cosine and Inverse Tangent

As in the case of the inverse sine function

$$\left. \begin{array}{l} y = \cos^{-1} x \text{ iff } \cos y = x \\ y = \tan^{-1} x \text{ iff } \tan y = x \end{array} \right\}$$

Note that,

$$d_{\cos^{-1} x} = [-1, 1], r_{\cos^{-1} x} = [0, \pi]$$

$$d_{\tan^{-1} x} = (-\infty, \infty), r_{\tan^{-1} x} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Also, note that $f(x) = \cos x$ and $f^{-1}(x) = \cos^{-1} x$

$$f^{-1}(f(x)) = x \text{ for } 0 \leq x \leq \pi$$

and $f(f^{-1}(x)) = x$ for $-1 \leq x \leq 1$

Similarly, for $f(x) = \tan x$ and $f^{-1}(x) = \tan^{-1} x$

$$f^{-1}(f(x)) = x \text{ for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

and $f(f^{-1}(x)) = x$ for $-\infty < x < \infty$

EX: Find exact value for $\cos^{-1}(\cos(-\frac{5\pi}{3}))$.
We know that $\cos^{-1}(\cos(-\frac{5\pi}{3})) = -\frac{5\pi}{3}$, but

$-\frac{5\pi}{3}$ is not in $[0, \pi]$. But $\frac{\pi}{3}$, an equivalent angle of $-\frac{5\pi}{3}$ is in the interval. Thus, $\cos(-\frac{5\pi}{3}) = \cos\frac{\pi}{3}$.

Finally, $\cos^{-1}(\cos(-\frac{5\pi}{3})) = \frac{\pi}{3}$

• Find Approximate Values for Inverse Functions MWF 12, F19

EX: Find the angles to nearest tenth of a degree and nearest hundredth of a radian.

(a) $\sin^{-1}\left(\frac{1}{3}\right) = \underline{19.5^\circ} = \underline{0.34}$

(b) $\cos^{-1}(0.058) = \underline{86.7^\circ} = \underline{1.51}$

TO BE CONT'D - 3.1 Continued

(c) $\tan^{-1}(1.2) = \underline{50.2^\circ} = \underline{0.88}$

(d) $\cos^{-1}(1.2) = \underline{\text{domain error}}$ because

$$\text{domain of } \cos^{-1} x = [-1, 1]$$

• Find An Inverse Function and Its Domain

EX: (a) Find the inverse, g^{-1} , of
 $g(x) = \cos(x+2) + 1, -2 \leq x \leq \pi - 2$

$$y = \cos(x+2) + 1$$

$$x = \cos(y+2) + 1$$

$$x-1 = \cos(y+2) \Rightarrow \cos^{-1}(x-1) = y+2$$

$$\cos^{-1}(x-1) - 2 = y$$

$$\Rightarrow g^{-1}(x) = \cos^{-1}(x-1) - 2$$

(b) Find the domain and range of g^{-1} .

We know that for g^{-1} ,

$$-1 \leq x-1 \leq 1$$

$$0 \leq x \leq 2$$

$$\Rightarrow d_{g^{-1}} = \underbrace{\{x \mid 0 \leq x \leq 2\}}_{\text{Range of } g} = [0, 2]$$

For g to be 1-1, $d_g = [-2, \pi - 2]$

Then, $r_{g^{-1}} = [-2, \pi - 2]$

- Solving Simple Equations Containing Inverse Trig. Functions

EX: Solve the given equation and state the solution set in exact form,

$$3 \sin^{-1} x = \pi$$

$$\sin^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow \sin \frac{\pi}{3} = x$$

$$\frac{\sqrt{3}}{2} = x$$

$$\text{Sol. Set} = \left\{ \frac{\sqrt{3}}{2} \right\}$$