MWF 12, F19

3.3 B Trig. Equations, Part II

- Solving Equations in Quadratic Form

(See A.4 in e-book to review how to solve quadratic equations, as needed.)

TO BE CONT'D

EX: Solve $2\sin^2\theta - 3\sin\theta + 1 = 0$, in $[0, 2\pi)$.

Factor the left side: $(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$ OR $\sin\theta - 1 = 0$

$2\sin\theta = 1$ $\qquad$ $\sin\theta = 1$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\Rightarrow \theta = \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$, Sol. Set $= \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$

EX: Solve $2\cos^2\theta + \cos\theta - 1 = 0$, θ in $[0, 2\pi)$.

Factor left side: $(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$ OR $\cos\theta + 1 = 0$

$2\cos\theta = 1$ $\qquad$ $\cos\theta = -1$

$\cos\theta = \frac{1}{2}$ $\qquad$ $\Rightarrow \theta = \pi$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Sol. Set $= \left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

- ## Solving Trig. Equations Using Identities

EX: Solve $\sin^2 x - \sin x = \cos^2 x$, x in $[0, 2\pi)$.

Using Identity: $\sin^2 x - \sin x = 1 - \sin^2 x$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$2\sin x + 1 = 0$ OR $\sin x - 1 = 0$

$2\sin x = -1$ $\qquad$ $\sin x = 1$

$\sin x = -\frac{1}{2}$ $\qquad$ $\Rightarrow x = \frac{\pi}{2}$

$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$, Sol. Set $= \underbrace{\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$

EX: Solve $4(1 + \sin\theta) = \cos^2\theta$, $0 \le \theta < 2\pi$.

$$4 + 4\sin\theta = \cos^2\theta$$

Using Identity: $4 + 4\sin\theta = 1 - \sin^2\theta$

$$\sin^2\theta + 4\sin\theta + 3 = 0$$

$$(\sin\theta + 3)(\sin\theta + 1) = 0$$

$\sin\theta + 3 = 0$ OR $\sin\theta + 1 = 0$

$\sin\theta = -3$ $\qquad$ $\sin\theta = -1$

$\Rightarrow \theta = \underline{\text{none}}$ $\qquad$ $\Rightarrow \theta = \frac{3\pi}{2}$

Sol. Set $= \underbrace{\left\{\frac{3\pi}{2}\right\}}$

- Solve Trig. Equations Linear in Sine and Cosine

EX: Solve $\cos\theta + 1 = \sin\theta$, θ in $[0, 2\pi)$.

We can square both sides so that we can use a Pythagorean identity directly. However, squaring both sides of any equation may introduce extraneous solutions. Therefore, we must check every solution using the original eq.

$$(\cos\theta + 1)^2 = \sin^2\theta$$

$$\cos^2\theta + 2\cos\theta + 1 = \sin^2\theta$$

$$\cos^2\theta + 2\cos\theta + 1 = 1 - \cos^2\theta$$

$$2\cos^2\theta + 2\cos\theta = 0$$

$$2\cos\theta(\cos\theta + 1) = 0$$

$2\cos\theta = 0$ OR $\cos\theta + 1 = 0$

$\cos\theta = 0$ $\quad$ $\cos\theta = -1$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\Rightarrow \theta = \pi$

MUST CHECK using $\cos\theta + 1 = \sin\theta$

For $\theta = \frac{\pi}{2}$, $\quad \cos\frac{\pi}{2} + 1 \overset{?}{=} \sin\frac{\pi}{2}$

✓ $\quad 0 + 1 \mid$

$1 = 1 \Rightarrow$ True

For $\theta = \frac{3\pi}{2}$, $\quad \cos\frac{3\pi}{2} + 1 \overset{?}{=} \sin\frac{3\pi}{2}$

✗ $\quad 0 + 1 \mid$

$1 \overset{?}{=} -1 \Rightarrow$ False

For $\theta = \pi$, $\quad \cos\pi + 1 \overset{?}{=} \sin\pi$

✓ $\quad -1 + 1 \mid$

$0 = 0 \Rightarrow$ True

Sol. Set $= \left\{\frac{\pi}{2}, \pi\right\}$

Class Practice Example: Solve

$\cos^2\theta - \sin^2\theta + \sin\theta = 0$, θ in $[0, 2\pi)$.

$1 - \sin^2\theta - \sin^2\theta + \sin\theta = 0$

$1 - 2\sin^2\theta + \sin\theta = 0$

$0 = 2\sin^2\theta - \sin\theta - 1$

$0 = (2\sin\theta + 1)(\sin\theta - 1)$

$2\sin\theta + 1 = 0$ OR $\sin\theta - 1 = 0$

$2\sin\theta = -1$ $\quad \sin\theta = 1$

$\sin\theta = -\frac{1}{2}$ $\quad \Rightarrow \theta = \frac{\pi}{2}$

$\Rightarrow \theta = \frac{11\pi}{6}, \frac{7\pi}{6}$

Sol. Set $= \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$