3.4 Trigonometric Identities

MWF 12, F19

Def - Two functions f and g are identically equal if $f(x) = g(x)$ for every x-value in their domains. Such an equation is called an identity.

Def - an equation that is not an identity is called a conditional equation.

EX: Use Algebra / Fundamental Identities to simplify each expression.

(a) $$\frac{\cot x}{\csc x} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} = \underbrace{\cos x}$$

(b) $$\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$$

$$= \frac{1-\cos x}{(1+\cos x)(1-\cos x)} + \frac{1+\cos x}{(1+\cos x)(1-\cos x)}$$

$$= \frac{1-\cos x+1+\cos x}{(1+\cos x)(1-\cos x)} = \frac{2}{1-\cos^2 x}$$

$$= \underbrace{\frac{2}{\sin^2 x}}$$

- <u>General Guidelines for Establishing/ Proving Identities</u>

The <u>goal</u> of a *proof* is to change <u>one</u> side of the identity to make it look like the <u>other</u> side.

(1) <u>Start</u> with the <u>more</u> complicated side.

(2) Add/Subtract any fractions on the side we choose.

(3) Re-write any trig. functions in terms of sine/cosine, if possible.

(4) Use the Fundamental Identities, factoring, etc., to simplify the expression.

(5) Keep in mind the <u>goal</u>. If one approach leads to a dead end, try another way, ...

EX: Establish / Prove each identity.

(a) $\csc x = \cos x \cot x + \sin x$.

RS $\cos x \cot x + \sin x = \cos x \cdot \dfrac{\cos x}{\sin x} + \sin x$

$$= \frac{\cos^2 x}{\sin x} + \sin x = \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x} = \underline{\csc x} \quad \text{LS}$$

(b) $\sec\beta = \tan\beta + \dfrac{\cos\beta}{1+\sin\beta}$.

RS $\tan\beta + \dfrac{\cos\beta}{1+\sin\beta} = \dfrac{\sin\beta}{\cos\beta} + \dfrac{\cos\beta}{1+\sin\beta}$

$$= \frac{\sin\beta(1+\sin\beta)}{\cos\beta(1+\sin\beta)} + \frac{\cos^2\beta}{\cos\beta(1+\sin\beta)}$$

$$= \frac{\sin^{\cancel{2}}\beta + \sin^2\beta + \cos^2\beta}{\cos\beta(1+\sin\beta)} = \frac{\sin\beta + 1}{\cos\beta(1+\sin\beta)}$$

$$= \frac{1}{\cos\beta} = \underline{\sec\beta} \quad \text{LS}$$

(c) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = 2\sec x.$

LS $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x}$

$$= \frac{\cos^2 x}{(1+\sin x)\cos x} + \frac{(1+\sin x)^2}{(1+\sin x)\cos x}$$

$$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$$

$$= \frac{1+1+2\sin x}{(1+\sin x)\cos x}$$

$$= \frac{2+2\sin x}{(1+\sin x)\cos x} = \frac{2(1+\sin x)}{(1+\sin x)\cos x}$$

$$= \frac{2}{\cos x} = \underline{2\sec x} \quad \text{RS}$$

(d) $\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} + 1 = 2\sin^2\theta$

LS $\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} + 1$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}} + 1$

$= \dfrac{\dfrac{\sin^2\theta}{\sin\theta\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta\cos\theta}}{\dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}} + 1$

$= \dfrac{\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}} + 1$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} \cdot \dfrac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta} + 1$

$= \sin^2\theta - \cos^2\theta + 1$

$= \sin^2\theta - (1 - \sin^2\theta) + 1$

$= \sin^2\theta - 1 + \sin^2\theta + 1$

$= \underline{2\sin^2\theta}$ RS