3.3 A Continued

MWF 12, F19

EX: Solve $\tan^2\theta = \frac{1}{3}$, θ is in $[0, 2\pi)$.

$$\sqrt{\tan^2\theta} = \pm\sqrt{\frac{1}{3}\cdot\frac{3}{3}}$$

$$\tan\theta = \pm\frac{\sqrt{3}}{3}$$

$\tan\theta = \frac{\sqrt{3}}{3}$ or $\tan\theta = -\frac{\sqrt{3}}{3}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{7\pi}{6}$ $\qquad$ $\Rightarrow \theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

$$\text{Sol. Set} = \underbrace{\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$$

EX: Solve $\cos(2\theta) = -\frac{1}{2}$, θ in $[0, 2\pi)$.

$$\Rightarrow 2\theta = \frac{2\pi}{3} + 2\pi n, \quad 2\theta = \frac{4\pi}{3} + 2\pi n$$

where n is an integer.

$n=0$, $2\theta = \frac{2\pi}{3} + 0$

$\theta = \frac{\pi}{3}$ ✓

$n=1$, $2\theta = \frac{2\pi}{3} + 2\pi$

$= \frac{2\pi}{3} + \frac{6\pi}{3}$

$2\theta = \frac{8\pi}{3}$

$\theta = \frac{4\pi}{3}$ ✓

$n=2$, $2\theta = \frac{2\pi}{3} + 4\pi$

$= \frac{2\pi}{3} + \frac{12\pi}{3}$

$2\theta = \frac{14\pi}{3}$

X $\theta = 7\pi/3 > 2\pi$

$n=0$, $2\theta = \frac{4\pi}{3} + 0$

$\theta = \frac{2\pi}{3}$ ✓

$n=1$, $2\theta = \frac{4\pi}{3} + 2\pi$

$= \frac{4\pi}{3} + \frac{6\pi}{3}$

$2\theta = \frac{10\pi}{3}$

$\theta = \frac{5\pi}{3}$ ✓

$$\text{Sol. Set} = \underbrace{\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}}$$

EX: Solve $\tan\left(\frac{\theta}{2}+\frac{\pi}{3}\right)-1=0$, θ in $[0,2\pi)$.

$$\text{let } x=\left(\frac{\theta}{2}+\frac{\pi}{3}\right)$$

$$\tan x = 1$$

$$\Rightarrow x=\frac{\pi}{4}+k\pi, \text{ where } k \text{ is an integer}$$

$$\Rightarrow \frac{\theta}{2}+\frac{\pi}{3}=\frac{\pi}{4}+k\pi, \quad LCD=12$$

$$6\theta+4\pi=3\pi+12k\pi$$

$$6\theta=-\pi+12k\pi$$

$$\theta=-\frac{\pi}{6}+2k\pi$$

For $k=1$, $\theta=-\frac{\pi}{6}+2\pi=-\frac{\pi}{6}+\frac{12\pi}{6}=\frac{11\pi}{6}$ ✓

$$\text{Sol. Set}=\left\{\frac{11\pi}{6}\right\}$$

- <u>Solving Equations Using Inverse Functions</u>

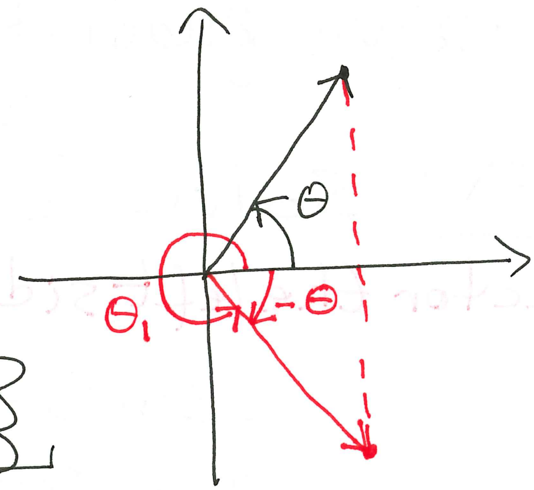
For some equations, it is difficult or impossible to find exact solutions. In such cases we use a calculator using inverse trig. functions. However, always keep in mind that inverse trig. functions exist from restricting the domain of the corresponding trig. functions.

EX: Solve $2+\cos\theta = 2.6$, for θ in $0 \le \theta < 2\pi$. Approx. sol's. to 3 decimals.

$$\cos\theta = 0.6$$

$$\Rightarrow \theta = \cos^{-1} 0.6 \quad (\text{mode in } \underline{\text{radians}})$$

$$\theta = 0.9273$$



or $\theta_1 = 2\pi - 0.9273$

$= 5.356$

Sol. Set $= \underline{\{0.927, 5.356\}}$

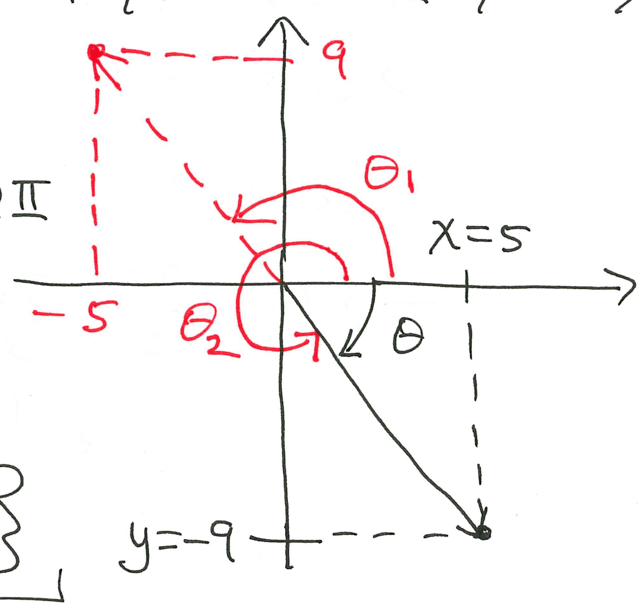
EX: Solve $5\tan\theta + 9 = 0$, for θ in $[0, 2\pi)$. Approx. the sol's to 3 decimals.

$$5\tan\theta = -9$$

$$\tan\theta = -\frac{9}{5} = \frac{y}{x} = \frac{-9}{5}$$

$$\Rightarrow \theta = \tan^{-1}\left(-\frac{9}{5}\right)$$

$\theta = -1.0637$, not in $[0, 2\pi)$.



However, $\theta_1 = \pi - 1.0637$

$\theta_1 = 2.078$ in QII

Also, $\theta_2 = 2\pi - 1.0637$

$= 5.219$ in QIV

Sol. Set $= \underline{\{2.078, 5.219\}}$