Infinite Limits and Vertical Asymptotes

ex.2 $f(x) = \frac{1}{x^2}$. **Examine the behavior of this function.**

First, list some things you know about this function.

1. It is a rational function with domain $(-\infty, 0) \cup (0, \infty)$.

2. The function is undefined at $x = 0$.

3. $\frac{1}{x^2} > 0$ for all x in the domain. This means the range of f is $(0, \infty)$

We need to see what the function values are doing near $x = 0$ (the place where f is undefined)

Limit from the left at zero: $\lim_{x \to 0^-} \frac{1}{x^2} = +\infty$

Limit from the right at zero: $\lim_{x \to 0^+} \frac{1}{x^2} = +\infty$

To determine this without knowing what the graph looks like, you can determine some function values when x is very close to zero but to the left and to the right of zero.

x	$y = \frac{1}{x^2}$
-0.1	100
-0.01	10,000
-0.001	1,000,000
0	undefined
0.001	1,000,000
0.01	10,000
0.1	100

About the notation: When we write $\lim_{x \to c} f(x) = \infty$, we mean that the values of the function, $f(x)$, can be made as large as we like by taking x to be sufficiently close to c.

ex.3 $f(x) = \frac{1}{x}$. **Examine the behavior of this function.**

Notice that this function is defined for all real numbers x except zero. How does the function behave near zero?

Limit from the left at zero: $\lim_{x \to 0^-} \frac{1}{x} = -\infty$.

Think about it this way. When x is very small and negative, $\frac{1}{x}$ will be very large and negative.

Limit from the right at zero: $\lim_{x \to 0^-} \frac{1}{x} = +\infty$.

Think about it this way. When x is very small and positive, $\frac{1}{x}$ will be very large and positive.