One Sided Limits (Left and Right Hand Limits)

Notation:

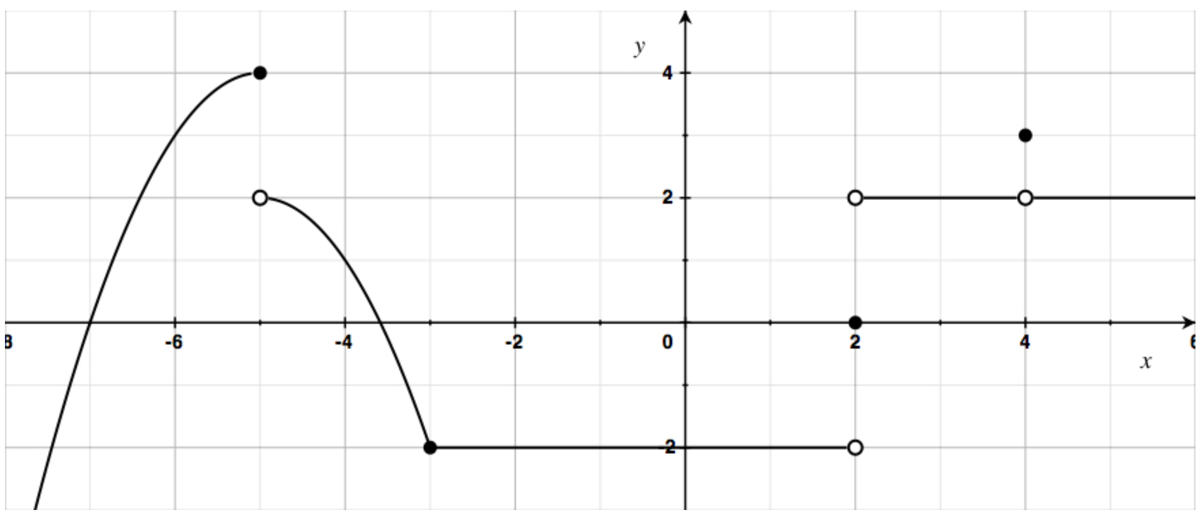
$$\lim_{x\to c^-} f(x) = L$$

means that as x gets very close to c from the left of c, the values of the function, get very close to L.

$$\lim_{x\to c^+} f(x) = L$$

means that as x gets very close to c from the right of c, the values of the function, get very close to L.

Example: The function $f(x)$ is graphed below, Determine each limit.



a. $\lim_{x\to -5^-} f(x) = 4$

b. $\lim_{x\to -5^+} f(x) = 2$

c. $\lim_{x\to -3^-} f(x) = -2$

d. $\lim_{x\to -3^+} f(x) = -2$

e. $\lim_{x\to 2^-} f(x) = -2$

f. $\lim_{x\to 2^+} f(x) = 2$

g. $\lim_{x\to 4^-} f(x) = 2$

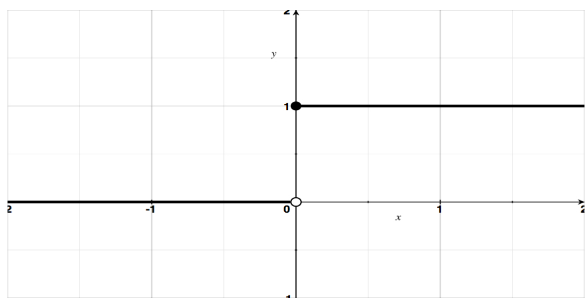
h. $\lim_{x\to 4^+} f(x) = 2$

Theorem: $\lim_{x \to c} f(x) = L \quad \Leftrightarrow \quad \lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$

This says that as $x \to c$, the limit of the function exists and is equal to L **if and only if** the limits from the left and from the right exist and are both equal to L.

ex.1 The Heaviside function (or Unit Step function). Think of turning on a light.

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$



The limit from the left at zero: $\lim_{x \to 0^-} f(x) = 0$

The limit from the right at zero: $\lim_{x \to 0^+} f(x) = 1$

Since the limits from the left and right are not equal, $\lim_{x \to 0} f(x)$ does not exist