MAT 1214: CALCULUS I
ONE SIDED LIMITS AND THE DEFINITION OF A LIMIT

(1) Sketch the graph and find the requested limits:

$$f(x) = \begin{cases} \sqrt{9-x^2} & 0 \le x < 3 \\ 3-x & 3 \le x < 6 \\ 3 & x = 6 \end{cases}$$

(a) $\lim_{x\to 0^+} f(x) =$__________.

(b) For $c \in (0,3)$, find $\lim_{x\to c} f(x) =$ __________.

(c) $\lim_{x\to 3^-} f(x) =$ __________.

(d) $\lim_{x\to 3^+} f(x) =$ __________.

(e) $\lim_{x\to 3} f(x) =$ __________.

(f) For $c \in (3,6)$, find $\lim_{x\to c} f(x) =$ __________.

(g) $\lim_{x\to 6^-} f(x) =$ __________.

(2) Find the following one-sided limits:

(a) $\lim_{x\to -1^+} \left(\dfrac{x}{x+3} \left(\dfrac{4x+8}{x^2+3x} \right) \right) =$ __________.

(b) $\lim_{x\to 3^-} \dfrac{\sqrt{5x}(x-3)}{|x-3|} =$ __________.

(c) $\lim_{h\to 0^-} \dfrac{\sqrt{3}-\sqrt{h^2+11h+3}}{h} =$ __________.

(3) Let $f(x) = \sqrt{x - x^2}$ for $0 \leq x \leq 1$.

(a) Find $L = \lim_{x \to 1^-} f(x)$.

$L =$ __________.

(b) Given a small number $\epsilon > 0$, find the largest $\delta > 0$ such that $|f(x) - L| < \epsilon$ when $1 - \delta < x < 1$.

$\delta =$ __________.

(c) Using a calculator, find the numerical value of δ (to at least 8 decimals) when $\epsilon = 0.01$.

$\delta =$ __________.